A SURROGATE MODEL TO BALANCE THE FLOW DISTRIBUTION IN COMPLEX PROFILE EXTRUSION DIES

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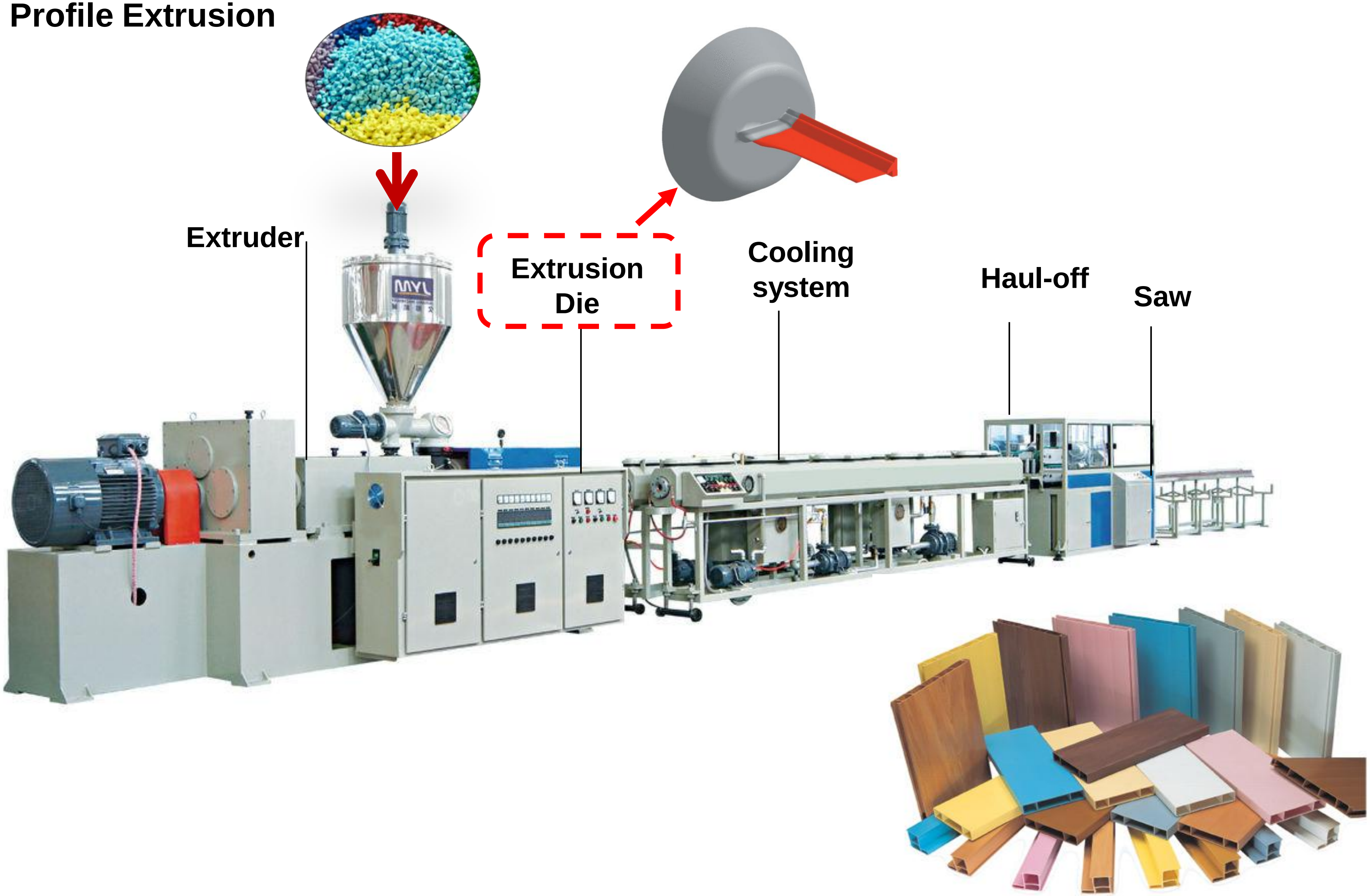
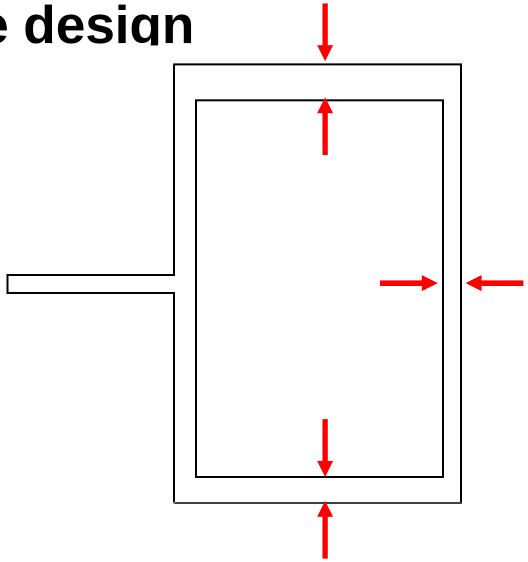
- **Motivation**
- **Solver Code**
- **Surrogate flow balance model**
  - ***L* and T shaped modular geometries.**
  - **Numerical runs**
  - **Surrogate model**
- **Die design methodology**
- **Assessment**
- **Conclusion**

Profile Extrusion



Profile Extrusion die design



Outlet profile cross section with different wall thicknesses

Unbalanced flow

Balanced flow

Nóbrega, J.M., Carneiro, O.S., Pinho, F.T., Oliveira, P.J. – Flow Balancing in Extrusion Dies for Thermoplastic Profiles. Part **III**: Experimental Assessment, **International Polymer Processing** Vol 19 (2004), p. 225-235;

Complex Profile Extrusion die design

- Flow balancing is very difficult
- High resource consumption (trial-and-error design approaches)
- highly dependent on designer's experience
- No usage of numerical tools in many extrusion companies

Need for useful design methodology for the extrusion die designers with no access to numerical modeling tools.

(**surrogate model:** obtained by a detailed numerical study performed with the aid of → OpenFOAM

Steady state non-isothermal flow of an incompressible GNF

- Mass conservation

$$\frac{\partial u_i}{\partial x_i} = 0$$

- Linear Momentum conservation

$$\frac{\partial p u_j u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x j}$$

$$\tau_{ij} = \eta(\dot{\gamma}) \ (\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$$

- Energy equation

$$\frac{\partial (u_i T)}{\partial x_i} - DT \frac{\partial}{\partial x_i}(\frac{\partial T}{\partial x_i}) = \frac{1}{c} (\tau_{ij} \frac{\partial u_i}{\partial x_j})$$

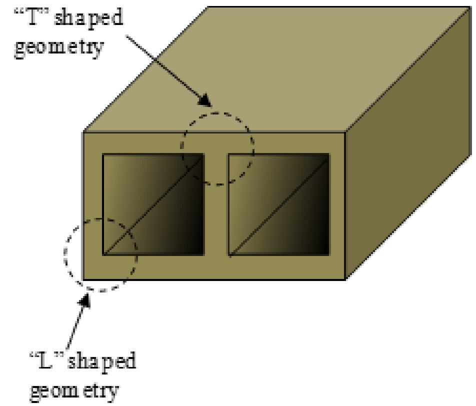
- Constitutive equation

$$\eta \ \dot{\gamma},T \ = F \ \dot{\gamma} \times H \ T \qquad H \ T$$

$$F \ \dot{\gamma} \ = \eta_\infty + \frac{\eta_0 - \eta_\infty}{1 + \ \lambda\dot{\gamma} \ ^{2} \ ^{\frac{1-n}{2}}} \qquad H \ T \ = \exp\left[\alpha\left(\frac{1}{T} - \frac{1}{T_\alpha}\right)\right]$$

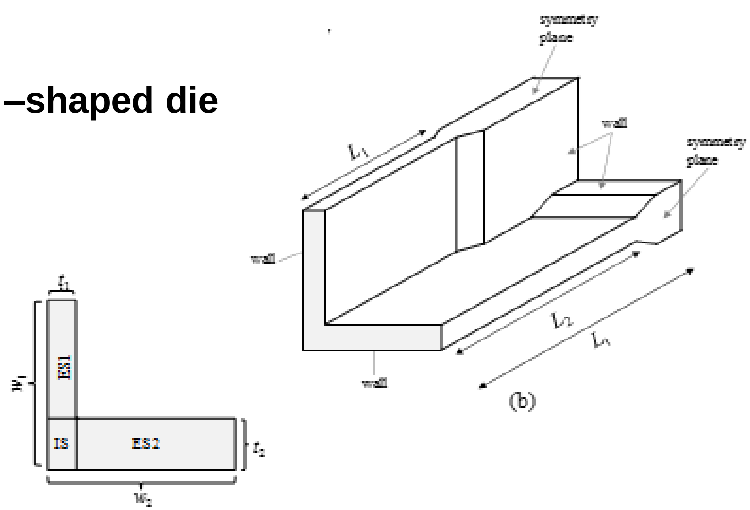
SimpleFoam + $\eta \ T, \dot{\gamma}$ + **Energy equation**

L and T shaped modular geometries

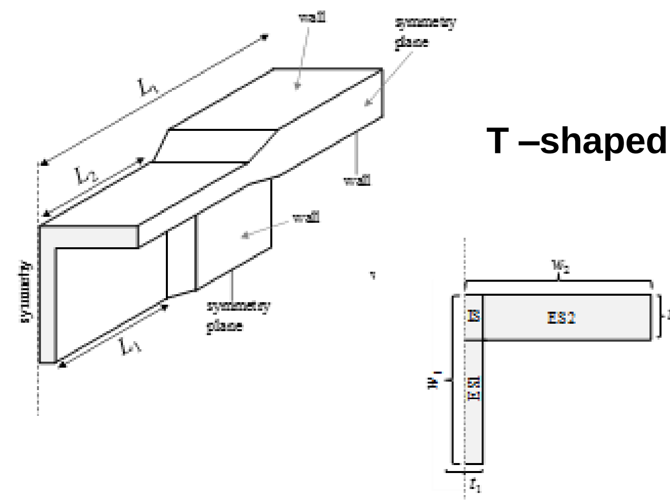


Numerical studies were performed with L and T shaped profiles.

L –shaped die



T –shaped die



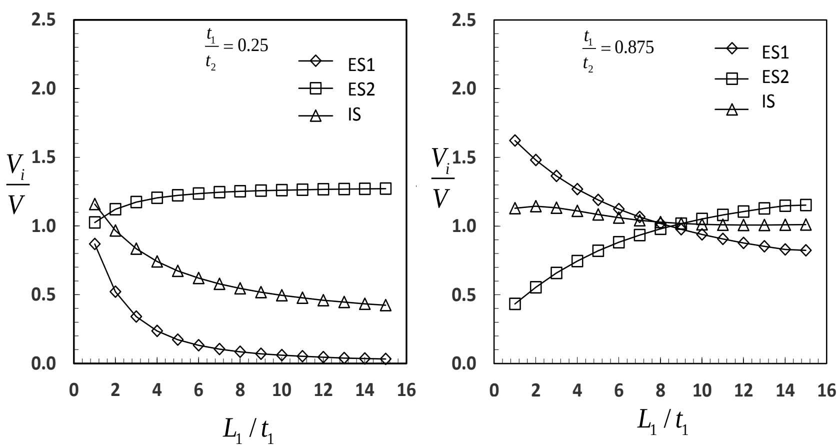
The outlet cross section is divided in Elemental Sections (ESs) and Intersection Sections (ISs) to monitor the flow for uniform distribution in all the ESs, Since flow in ISs is not controllable.

Numerical runs

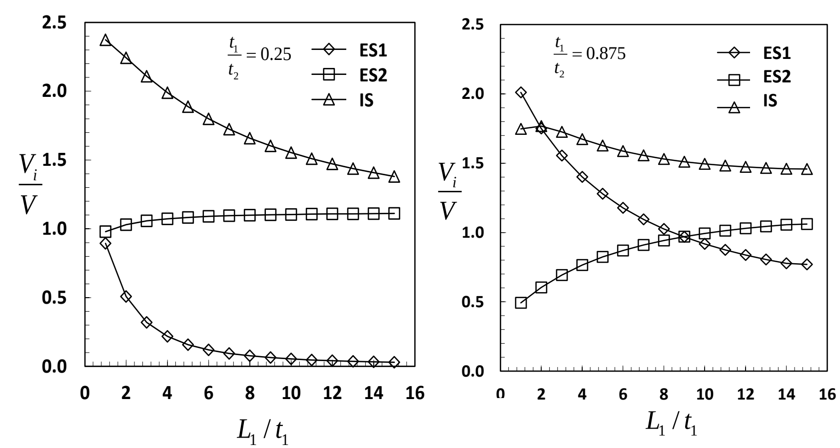
L_2/t_2	t_2 [mm]	L_2 [mm]	t_1/t_2	t_1 [mm]
15	1	15	[0.25; 0.875]	[0.25; 0.875]
	2	30		[0.5; 1.75]
	4	60		[1; 3.5]
10	1	10	{0.375;0.625; 0.875}	{0.325;0.675; 0.875}
	2	20		{0.75;1.25; 1.75}
	4	40		{1.5;2.5; 3.5}
5	1	5	{0.375;0.625; 0.875}	{0.325;0.675; 0.875}
	2	10		{0.75;1.25; 1.75}
	4	20		{1.5;2.5; 3.5}

Flow modeling of polypropylene homopolymer extrusion grade was numerically studied, to obtain the surrogate flow balance models, more than 400 simulations were performed.

L –shaped die



T –shaped die



L_1/t_1 value for the intersection of ES1 and ES2 are monitored in each case.

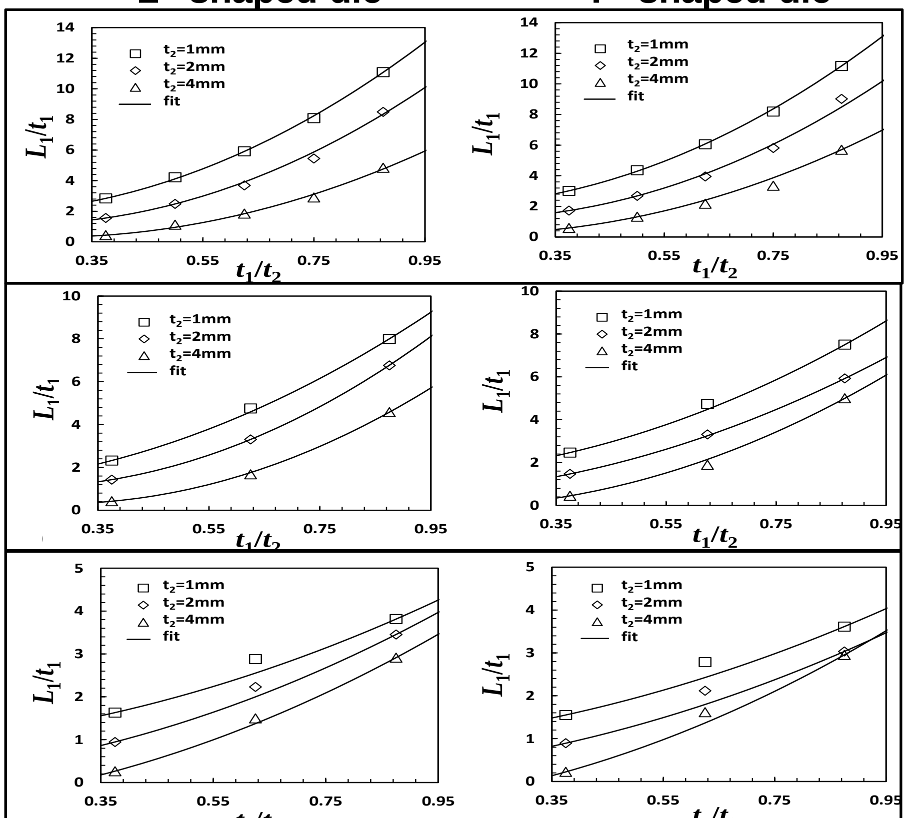
Surrogate flow balance model

Surrogate model

All the curves shows a *quadratic* growth of L_1/t_1. A fitting formula was devised with the dependence of the balanced flow point (L_1/t_1) on the thickness ratio (t_1/t_2).

$$\frac{L_1}{t_1} = a(t_2, L_2/t_2)\left[\frac{t_1}{t_2}\right]^2 + b\,(t_2, L_2/t_2)\left[\frac{t_1}{t_2}\right] + c\,(t_2, L_2/t_2)$$

L –shaped die **T –shaped die**



Mean relative errors, obtained with the fitting functions for the L and T die geometries.

	L_2/t_2	Mean Relative Error [%] t_2=1mm	t_2=2mm	t_2=4mm
***L*-die**	5	3.801	3.478	2.744
	10	1.150	0.001	1.654
	15	0.853	3.131	4.784
***T*-die**	5	4.306	5.038	5.163
	10	1.822	0.638	4.506
	15	0.794	2.942	5.496

Surrogate flow balance model

Surrogate model

$$\frac{L_1}{t_1} = a(t_2\,, L_2/t_2)\left[\frac{t_1}{t_2}\right]^2 + b\,(t_2\,, L_2/t_2)\left[\frac{t_1}{t_2}\right] + c\,(t_2\,, L_2/t_2)$$

L – die

$a=x_1(t_2)^2+y_1(t_2)+z_1$

$b=x_2(t_2)^2+y_2(t_2)+z_2$

$c=x_3(t_2)^2+y_3(t_2)+z_3$.

T – die

$a=u_1(t_2)^2+v_1(t_2)+w_1$

$b=u_2(t_2)^2+v_2(t_2)+w_2$

$c=u_3(t_2)^2+v_3(t_2)+w_3$

***L*-die**

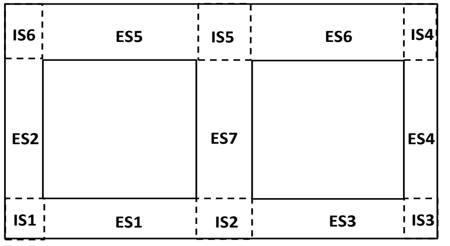
	Parameters	Quadratic equations
a	x_1	$0.03734510(L_2/t_2)^2 - 0.80169016\,(L_2/t_2) + 2.93937832$
	y_1	$-0.24939070(L_2/t_2)^2 + 5.08338350(L_2/t_2) - 18.26560500$
	z_1	$0.23746280(L_2/t_2)^2 - 3.38520534(L_2/t_2) + 12.91528668$
b	x_2	$-0.04051703(L_2/t_2)^2 + 0.91053283(L_2/t_2) - 3.54060001$
	y_2	$0.27547970(L_2/t_2)^2 - 6.07985350(L_2/t_2) + 23.52837000$
	z_2	$-0.30383447(L_2/t_2)^2 + 6.00266366(L_2/t_2) - 21.44919999$
c	x_3	$0.01189827(L_2/t_2)^2 - 0.25518200(L_2/t_2) + 1.11254833$
	y_3	$-0.07780220(L_2/t_2)^2 + 1.65860100(L_2/t_2) - 7.51204500$
	z_3	$0.08473653(L_2/t_2)^2 - 1.65247800(L_2/t_2) + 8.05443667$

***T*-die**

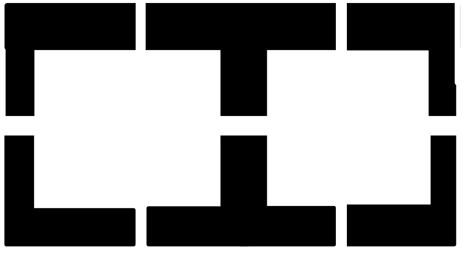
	Parameters	Quadratic equations
a	u_1	$-0.03283900(L_2/t_2)^2 + 0.60375660(L_2/t_2) - 2.09687500$
	v_1	$0.12601310(L_2/t_2)^2 - 2.43891870(L_2/t_2) + 8.89061200$
	w_1	$-0.04083970(L_2/t_2)^2 + 2.16125150(L_2/t_2) - 7.21692400$
b	u_2	$0.03474026(L_2/t_2)^2 - 0.60500330(L_2/t_2) + 2.17478900$
	v_2	$-0.14470624(L_2/t_2)^2 + 2.42773880(L_2/t_2) - 8.60696400$
	w_2	$0.05018524(L_2/t_2)^2 - 1.12527240(L_2/t_2) + 5.42178600$
c	u_3	$-0.00788694(L_2/t_2)^2 + 0.14490350(L_2/t_2) - 0.44020200$
	v_3	$0.03579614(L_2/t_2)^2 - 0.65609770(L_2/t_2) + 1.45577200$
	w_3	$-0.02011246(L_2/t_2)^2 + 0.49551710(L_2/t_2) - 0.30281100$

Die design methodology

In a typical complex profile design problem there are more restrictions than variables (for example, conflicting values of lengths of neighboring ESs)



Complex geometry division into IS and ES



partition into L and T shaped geometries

The surrogate model was not devised in a way to define all the length of the ESs at a time.

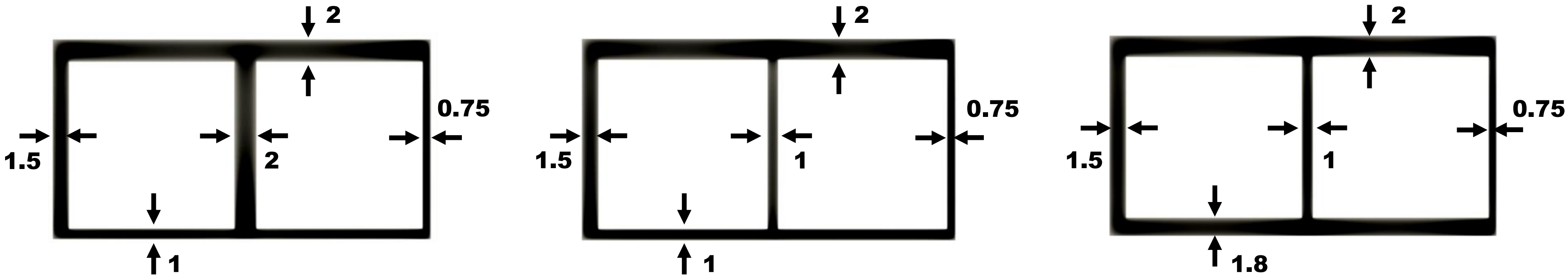
For that, two different optimization methods were devised to guide the design of profile extrusion dies based on the information provided by the model:

Average Method: based on averaging the lengths of each section, a flow chart was developed to guide this process.

Minimum Method: based on the minimization of the differences between the length values given by the fitting functions applied to each ES and the actual length imposed

Average Method is self-contained, while the Minimum Method requires the use of a software (in our case Mathematica) to find the solution of a system of equations

The die design methodology is assessed with three different complex geometries.



Geometry 1 **Geometry 2** **Geometry 3**

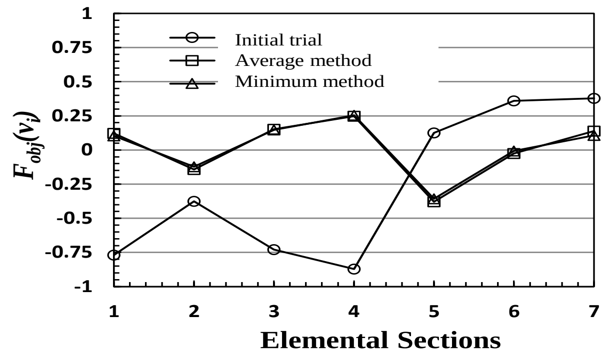
Objective function F_{obj}, calculated during the assessment process

$$F_{obj}(V_i) = \frac{\frac{V_i}{V_{obj}} - 1}{max\left(\frac{V_i}{V_{obj}}\,,\ 1\right)}$$

Assessment

Geometry 1

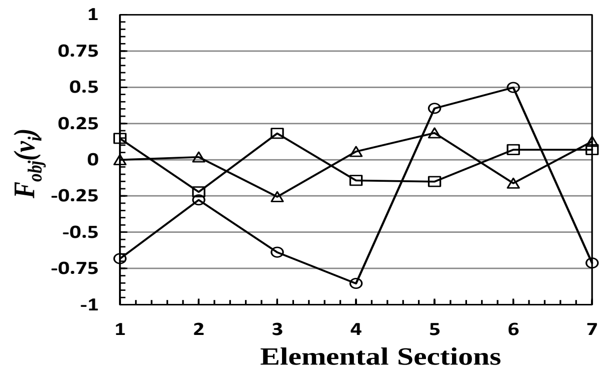
Elemental Sections	Thickness of ESs (mm)	Initial dimensions L/t	Initial dimensions L	Average method Final dimensions L/t	Average method Final dimensions L	Minimum method Final dimensions L/t	Minimum method Final dimensions L
ES4	0.75	15	11.25	1.53	1.15	1.5	1.125
ES1,ES3	1	15	15	2.77	2.77	2.73	2.73
ES2	1.5	15	22.5	5.87	8.81	5.58	8.37
ES7	2	15	30	15	30	16.73	37.19
ES5,ES6	2	15	30	15	30	12.91	25.82



(a)

Geometry 2

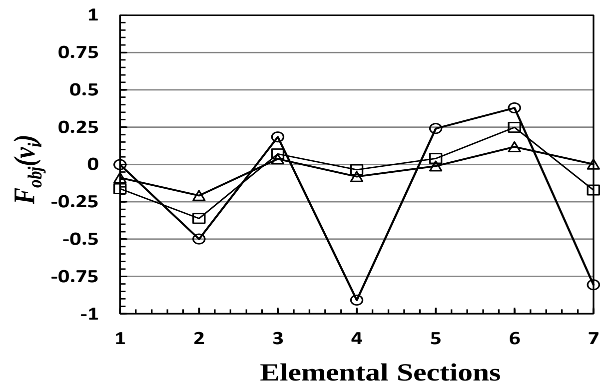
Elemental Sections	Thickness of ESs (mm)	Initial dimensions L/t	Initial dimensions L	Average method Final dimensions L/t	Average method Final dimensions L	Minimum method Final dimensions L/t	Minimum method Final dimensions L
ES4	0.75	15	11.25	1.53	1.15	1.43	1.07
ES7	1	15	15	2.68	2.68	2.61	2.61
ES1,ES3	1	15	15	2.77	2.77	3.1	3.1
ES2	1.5	15	22.5	5.87	8.81	5.9	8.85
ES5,ES6	2	15	30	15	30	17.36	34.72



(b)

Geometry 3

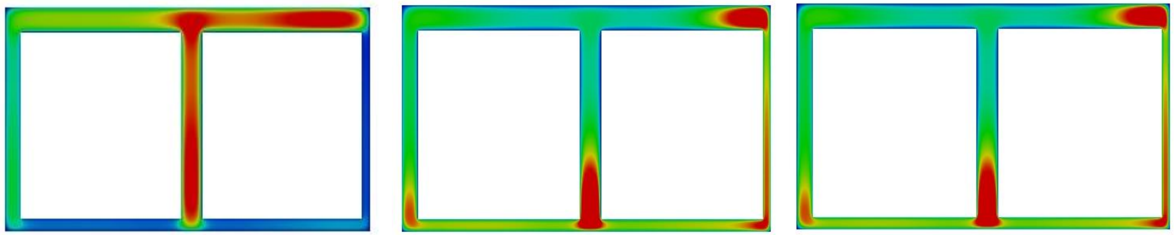
Elemental Sections	Thickness of ESs (mm)	Initial dimensions L/t	Initial dimensions L	Average method Final dimensions L/t	Average method Final dimensions L	Minimum method Final dimensions L/t	Minimum method Final dimensions L
ES4	0.75	15	11.25	1.81	1.36	1.46	1.095
ES7	1	15	15	3.12	3.12	2.21	2.21
ES2	1.5	15	22.5	6.9	10.35	4.74	7.1
ES1,ES3	1.8	15	27	15	27	7.12	12.82
ES5,ES6	2	15	30	15	30	9.742	19.48



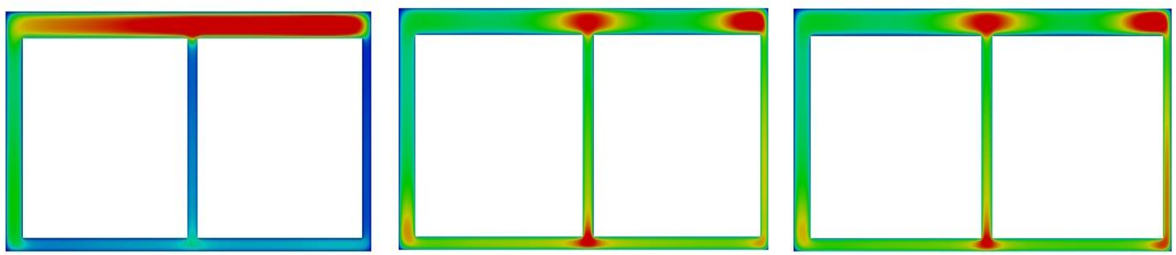
(c)

Assessment

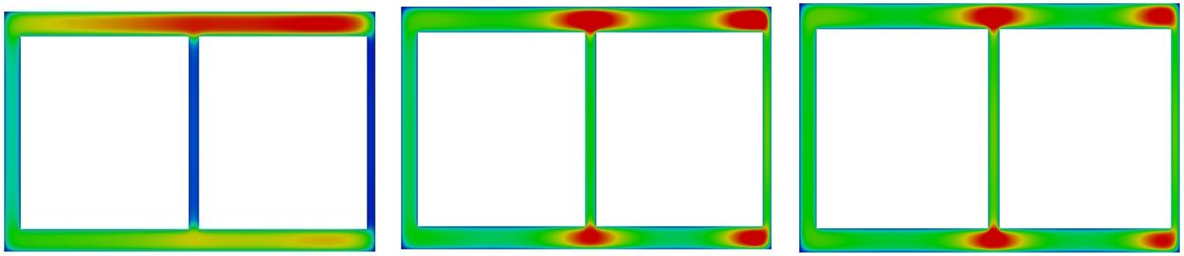
Geometry 1

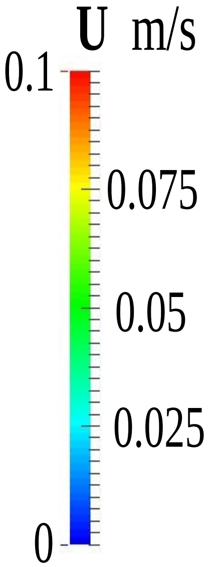


Geometry 2



Geometry 3





Assessment

Error (**L_2 norm and L_∞ norm**) obtained for the different methods

Geometry 1	L_2	L_∞
Initial trial	1.654	0.872
Average Method	0.592	0.379
Minimum Method	0.562	0.357

Geometry 2	L_2	L_∞
Initial trial	1.863	0.993
Average Method	0.427	0.223
Minimum Method	0.413	0.255

Geometry 3	L_2	L_∞
Initial trial	1.499	0.908
Average Method	0.553	0.362
Minimum Method	0.278	0.208

the Minimum Method provides the smallest error, since the overall influence of one section over the other is taken in to account.

Conclusion

- A new OpenFOAM solver able to cope with non-isothermal steady state flows of GNF was implemented.
- A surrogate model was developed by performing numerical studies with *L* and *T* shaped geometries.
- A novel methodology to balance the flow distribution in complex extrusion dies was proposed.
- The proposed methodology was assessed with three complex geometries.

Thanks for your attention!