Improvements in the OpenFOAM® numerical code for simulation of steady-state differential viscoelastic flows

Fernandes, C., De Araujo, M.S.B., Ferrás, L.L., Nóbrega, J.M.

i3N/Institute for Polymers and Composites
Minho University, Portugal

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Outline

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- For the definition of the final characteristics and quality of finished products in several industry sectors the viscoelastic flows are needed to be understood.
- The viscoelastic fluids behavior is complex, including viscous and elastic effects and highly non-linear phenomena.
- Strain rate dependent viscosity, presence of normal stress differences, relaxation phenomena and memory effects.

Outline

Alves et al., 2000

Developed a finite-volume method (FVM) which as an improved accuracy and enhanced convergence rate when using in conjunction with a **high-resolution scheme** to represent the stress derivatives in the constitutive equation. Calculations for the benchmark flow of an **upper convected Maxwell** fluid through a **4:1 plane contraction** are presented.

Alves et al., 2001

Accurate solutions are obtained for the inertialess plane **flow around a cylinder**. Two **high-resolution schemes** (MINMOD and SMART) are implemented to represent the convective terms in the constitutive equations for the **upper convected Maxwell** and **Oldroyd-B** fluids, and the resulting predictions of the drag coefficient on the cylinder are shown to be as accurate as existing finite-element method predictions.

Numerical Studies

Favero et al., 2010

Presented a numerical methodology based on the **split-stress tensor** approach using any differential constitutive equations. The proposed methodology was implemented in **OpenFOAM**®, a flexible open source CFD package.

Outline

Objectives

- Development of an OpenFOAM® solver for accurate and stable solution of viscoelastic fluids flows: using the DEVSS methodology proposed by Guénette and Fortin [1] and high-resolution differencing schemes.
- Verify the ability of the code to predict the flow characteristics in the benchmark problems 4:1 plane contraction and flow around a cylinder, and compare with the available results in the literature.

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Conservation of Mass

$$\nabla \cdot \mathbf{u} = 0$$

Conservation of Momentum

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{u}\mathbf{u} \right] = -\nabla p + \nabla \cdot \boldsymbol{\tau}$$

The total stress tensor is written as $\boldsymbol{\tau} = \boldsymbol{\tau}_S + \boldsymbol{\tau}_P$, where $\boldsymbol{\tau}_S = \eta_S \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T \right)$ and the polymeric contribution is given by a constitutive equation.

Equations II

Constitutive Equation (upper convected Maxwell)

$$\boldsymbol{\tau}_P + \lambda \overset{\triangledown}{\boldsymbol{\tau}}_P = \eta_P [\nabla \mathbf{u} + (\nabla \mathbf{u})^T]$$

with

$$\overset{\triangledown}{\boldsymbol{\tau}}_P = \frac{\partial \boldsymbol{\tau}_P}{\partial t} + \nabla \cdot (\mathbf{u} \boldsymbol{\tau}_P) - (\nabla \mathbf{u})^T \cdot \boldsymbol{\tau}_P - \boldsymbol{\tau}_P \cdot \nabla \mathbf{u}$$

For this model, $\boldsymbol{\tau}_S = 0$.

Discrete elastic-viscous stress splitting DEVSS

Additional elliptic term

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla\cdot(\rho\mathbf{u}\mathbf{u}) - \nabla\cdot\eta^{\star}\nabla\mathbf{u} = -\nabla p + \nabla\cdot\boldsymbol{\tau}_P - \nabla\cdot\eta^{\star}\nabla\mathbf{u}$$

The above described model was implemented in the software package OpenFOAM® by Favero et al. (2010) [2].

Pressure-velocity coupling

- Accomplished by segregated methods, in which the continuity equation is used to formulate an equation for the pressure.
- The resulting equation set is solved by a decoupled approach, using iterative algorithms with under-relaxation, such as SIMPLE.

Velocity-stress coupling

- Using the methodology presented by Guénette and Fortin (1995) [1] the extra-stress tensor is defined as:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_P - \alpha[\nabla \mathbf{u} + (\nabla \mathbf{u})^T]$$

- The resulting momentum equation reads as follows:

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) - \nabla \cdot \alpha \nabla \mathbf{u} = -\nabla p + \nabla \cdot \boldsymbol{\tau}$$

- There is no need to introduce the new variable $\boldsymbol{\tau}$ into the constitutive equation.

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Normalized Variable Approach (NVA) I

Definition

The convected variable ϕ is normalised as:

$$\tilde{\phi} = \frac{\phi - \phi_U}{\phi_D - \phi_U}$$

where the subscripts U and D indicate the upstream and downstream cells to cell P which is the cell immediately upstream of cell face 'f' under consideration.

Normalized Variable Approach (NVA) II

MINMOD scheme

$$\tilde{\phi}_f = \begin{cases} \frac{3}{2}\tilde{\phi}_C & \text{if } 0 \leq \tilde{\phi}_C < \frac{1}{2} \\ \frac{\tilde{\phi}_C + 1}{2} & \text{if } \frac{1}{2} \leq \tilde{\phi}_C \leq 1 \\ \tilde{\phi}_C & \text{if } \tilde{\phi}_C \notin [0, 1] \end{cases}$$

SMART scheme

$$\tilde{\phi}_f = \begin{cases} 3\tilde{\phi}_C & \text{if } 0 \leq \tilde{\phi}_C < \dfrac{1}{6} \\ \dfrac{3}{4}\tilde{\phi}_C + \dfrac{3}{8} & \text{if } \dfrac{1}{6} \leq \tilde{\phi}_C \leq \dfrac{5}{6} \\ 1 & \text{if } \dfrac{5}{6} < \tilde{\phi}_C \leq 1 \\ \tilde{\phi}_C & \text{if } \tilde{\phi}_C \notin [0,1] \end{cases}$$

Flux Limiters Approach (FLA) I

Relations between NVA and FLA

$$\psi(r) = \frac{\tilde{\phi}_f - \tilde{\phi}_C}{(1 - \tilde{\phi}_C)/2}$$

$$r = \frac{\tilde{\phi}_C}{1 - \tilde{\phi}_C}$$

Flux Limiters Approach (FLA) II

MINMOD scheme

$$\psi_{MINMOD}(r) = \begin{cases} 0 & \text{if } r \leq 0 \\ r & \text{if } 0 < r < 1 \\ 1 & \text{if } r \geq 1 \end{cases}$$

SMART scheme

$$\psi_{SMART}(r) = \begin{cases} 0 & \text{if } r \leq 0 \\ 4r & \text{if } 0 < r \leq \dfrac{1}{5} \\ \dfrac{3+r}{4} & \text{if } \dfrac{1}{5} < r < 5 \\ 2 & \text{if } r \geq 5 \end{cases}$$

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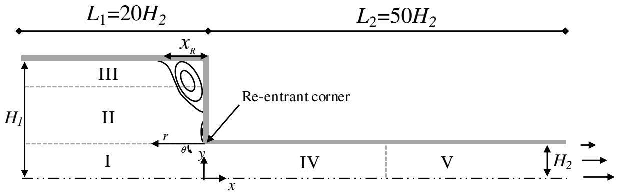
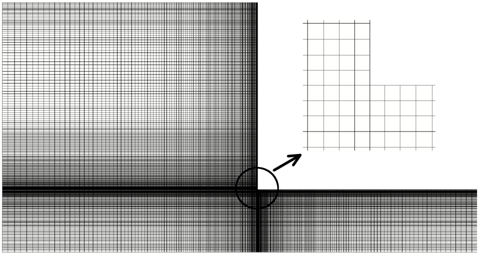


L_1=20H_2
L_2=50H_2
x_R
III
II
I
H_1
Re-entrant corner
r
θ
y
x
IV
V
H_2



Parameters

Upstream channel height H_1 0.02 m

Downstream channel height H_2 0.005 m

Downstream velocity U_2 0.005 m/s

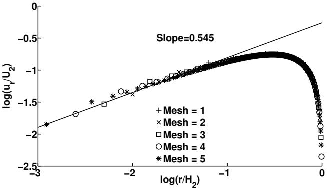
Fluid density ρ 100 kg/m^3

Polymeric viscosity η_P 0.25 $kg/m.s$

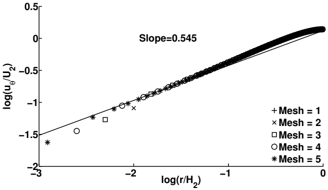
The Reynolds number defined on the basis of downstream channel quantities is therefore equal to $Re = \frac{\rho U_2 H_2}{\eta} = 0.01$.

The Deborah number also defined on the basis of downstream channel quantities $De = \frac{\lambda U_2}{H_2}$ was varied by changing the parameter λ from 0 to 5.

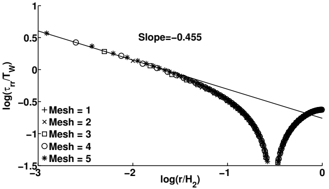
Newtonian asymptotic behavior I



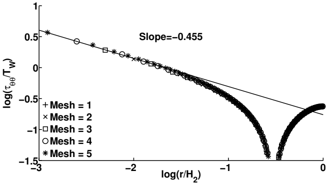
(a)



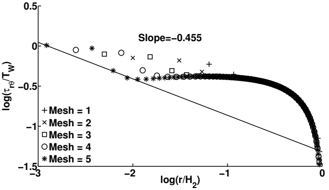
(b)


Slope=−0.455
log(τ_rr/T_W)
log(r/H_2)
Mesh = 1
Mesh = 2
Mesh = 3
Mesh = 4
Mesh = 5

(c)

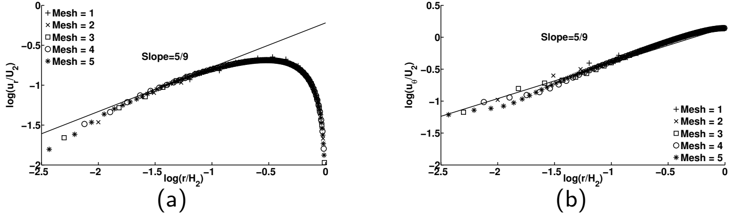

Slope=−0.455
log(τ_θθ/T_W)
log(r/H_2)
Mesh = 1
Mesh = 2
Mesh = 3
Mesh = 4
Mesh = 5

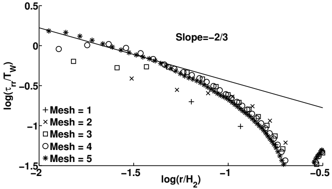
(d)


Slope=−0.455
log(τ_rθ/T_W)
log(r/H_2)
Mesh = 1
Mesh = 2
Mesh = 3
Mesh = 4
Mesh = 5

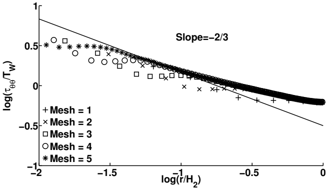
(e)

UCM asymptotic behavior I - De = 1

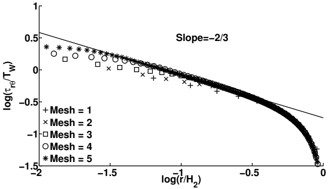



Slope=−2/3
log(τ_rr/T_W)
log(r/H_2)
+ Mesh = 1
× Mesh = 2
□ Mesh = 3
○ Mesh = 4
∗ Mesh = 5

(c)

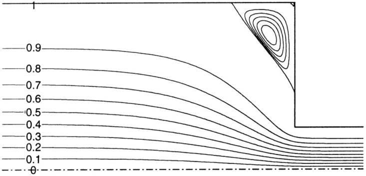

Slope=−2/3
log(τ_θθ/T_W)
log(r/H_2)
+ Mesh = 1
× Mesh = 2
□ Mesh = 3
○ Mesh = 4
∗ Mesh = 5

(d)

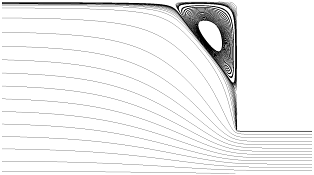

Slope=−2/3
log(τ_rθ/T_W)
log(r/H_2)
+ Mesh = 1
× Mesh = 2
□ Mesh = 3
○ Mesh = 4
∗ Mesh = 5

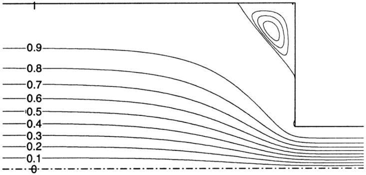
(e)

Streamlines and corner vortex size I

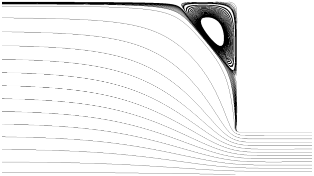


(a) $De = 0$

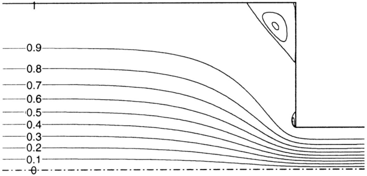

(b) $De = 0$



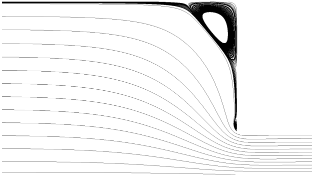
(c) $De = 1$


(d) $De = 1$

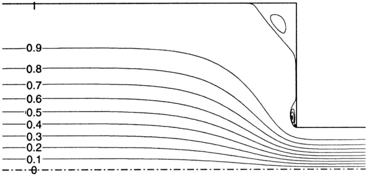
Streamlines and corner vortex size II



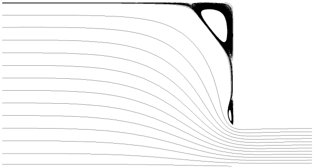
(e) $De = 2$



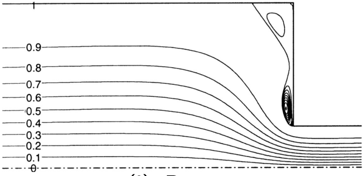
(f) $De = 2$



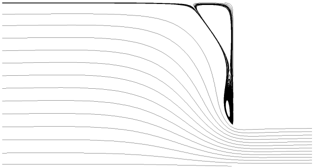
(g) $De = 3$



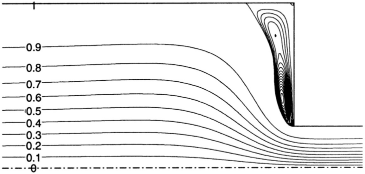
(h) $De = 3$



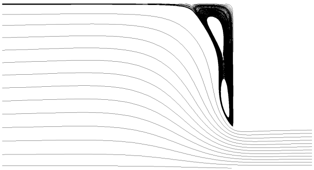
(i) $De = 4$



(j) $De = 4$



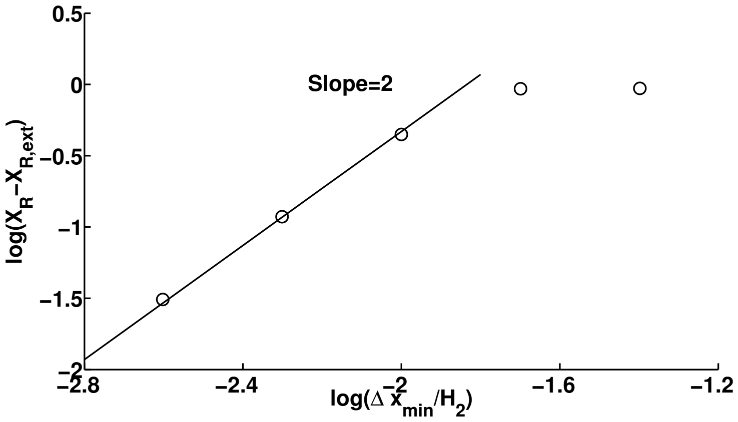
(k) $De = 5$



(l) $De = 5$

Streamlines and corner vortex size IV

Developed code							
De	Mesh 1	Mesh 2	Mesh 3	Mesh 4	Mesh 5	Extrapolated	Difference (%)
0	1.436	1.475	1.477	1.479	1.479	1.479	0.0003
1	1.374	1.378	1.349	1.330	1.321	1.314	0.6
2	1.301	1.285	1.176	1.113	1.083	1.056	2.5
3	1.305	1.290	1.054	0.928	0.881	0.854	3.2
4	1.402	1.396	1.014	0.803	0.735	0.702	4.7
5	1.530	1.524	1.037	0.709	0.622	0.591	5.3
Alves et al. (2000) [3]							
De	Mesh 1	Mesh 2	Mesh 3	Mesh 4	Extrapolated	Difference (%)	
0	1.472	1.488	1.494	1.495	1.496	0.1	
1	1.349	1.371	1.349	1.339	1.335	0.3	
2	1.631	1.259	1.154	1.118	1.105	1.2	
3	1.517	1.266	1.014	0.946	0.923	2.5	
4	1.644	1.337	0.987	–	0.87	13.4	
5	1.687	1.517	1.127	–	0.997	13	



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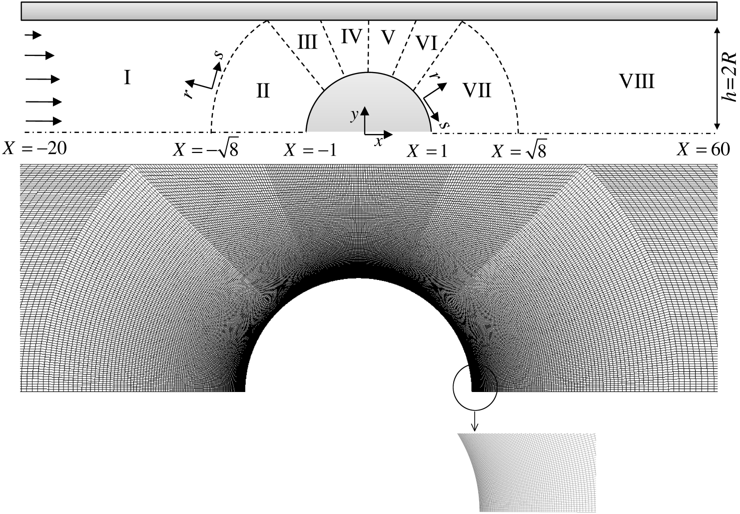

I
II
III
IV
V
VI
VII
VIII
r
s
y
x
h=2R
X = −20
X = −√8
X = −1
X = 1
X = √8
X = 60

Parameters

Channel height h 2 m

Cylinder radius R 1 m

Uniform velocity U 1 m/s

Fluid density ρ 100 kg/m^3

Polymeric viscosity η_P 10000 $kg/m.s$

The Reynolds number defined on the basis of the inlet bulk velocity, U, and the cylinder radius, R, is therefore equal to $Re = \frac{\rho U R}{\eta} = 0.01$.

The Deborah number also defined on the basis of the inlet bulk velocity, U, and the cylinder radius, R, is $De = \frac{\lambda U}{R}$ and was varied by changing the parameter λ from 0 to 0.9.

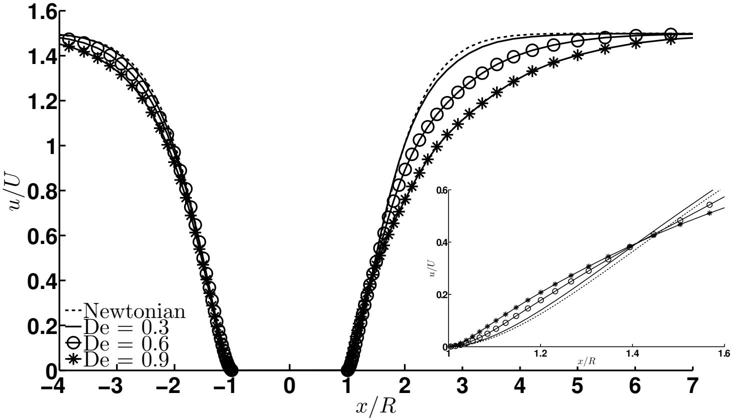
Drag coefficient

Definition

$$C_D = \frac{1}{\rho U^2 R} \int_S (\boldsymbol{\tau} - p\boldsymbol{I}) \cdot \boldsymbol{n} \cdot \hat{\boldsymbol{i}} dS$$

	Developed code			Alves et al. (2001) [4]			Errors (%)		
De	M30	M60	M120	M30	M60	M120	M30	M60	M120
0	132.071	132.435	132.503	132.23	132.342	132.369	0.1	0.1	0.1
0.3	107.956	108.435	108.891	–	108.515	108.614	–	0.1	0.3
0.6	93.142	92.828	93.197	–	92.277	92.298	–	0.6	1.0
0.9	87.182	87.244	87.242	–	87.395	87.218	–	0.2	0.03

Longitudinal velocity along the centreline



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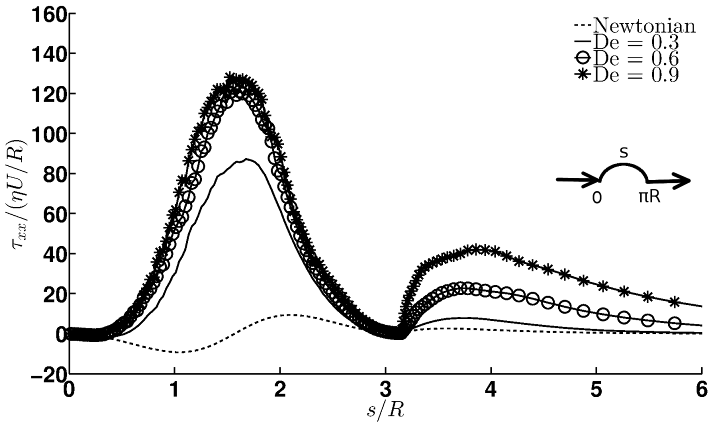
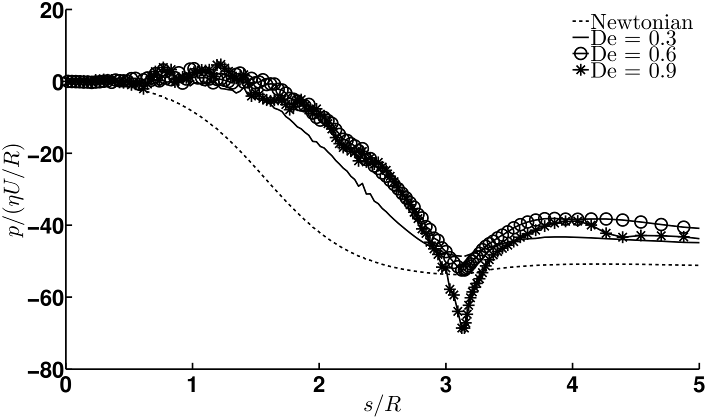
Normal stress τ_{xx} along cylinder wall and wake centreline



Pressure around the cylinder surface and along the wake centreline



Outline

Conclusions

- The newly improved solver was assessed in terms of performance and accuracy of the method, where high-order interpolation schemes were used for the convective terms.
- As expected, the predicted order of convergence of the method using the MINMOD scheme was found to be approximately 2.
- The presented results are in agreement with the literature for the benchmark test cases of the 4:1 plane contraction and flow around a cylinder.
- These results permits to validate the implemented modeling and the methodology used.
- Paper (in preparation): Numerical simulation of upper convected Maxwell fluid with a open-source general collocated finite-volume method

Outline

Future Work

- Perform studies involving integral viscoelastic models.
- Improvements in the speed of the calculations using a coupled approach.
- Development of special no-slip boundaries conditions.

References

[1] R. Guénette and M. Fortin, “A new mixed finite element method for computing viscoelastic flows,” *J. Non-Newton. Fluid Mech.*, vol. 60, pp. 27–52, 1995.

[2] J. Favero, A. Secchi, N. Cardozo, and H. Jasak, “Viscoelastic flow analysis using the software OpenFOAM and differential constitutive equations,” *J. Non-Newton. Fluid Mech.*, vol. 165, pp. 1625–1636, 2010.

[3] M. Alves, F. Pinho, and P. Oliveira, “Effect of a high-resolution differencing scheme on finite-volume predictions of viscoelastic flows,” *J. Non-Newtonian Fluid Mech.*, vol. 93, pp. 287–314, 2000.

[4] M. Alves, F. Pinho, and P. Oliveira, “The flow of viscoelastic fluids past a cylinder: finite-volume high-resolution methods,” *J. Non-Newtonian Fluid Mech.*, vol. 97, pp. 207–232, 2001.

Thank you for your attention