# DEVELOPMENT OF A SUB-GRID MODEL FOR SELF-AERATION


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# Air-entrainment

- Air-entrainment occurs naturally in most **free-surface turbulent** flows;

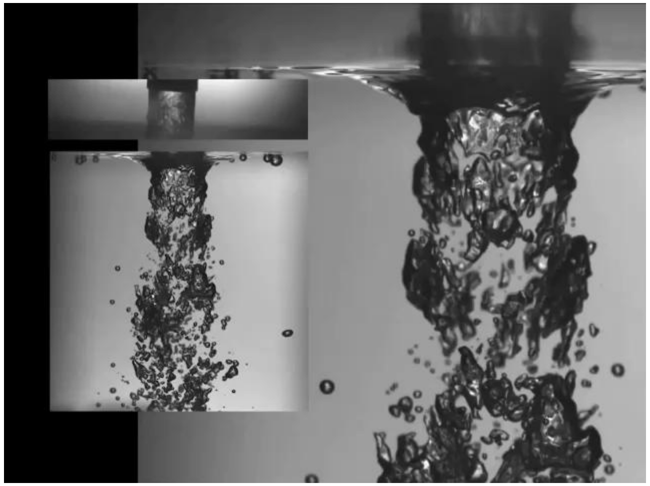
- **Importance to the flow characteristics:**
    - Changes the turbulent structure,
    - Density and compressibility are modified,
    - Increases the volume of the flow,

- **Consequences:**
    - River re-oxigenation;
    - Waste-water treatment;
    - Energy dissipation;
    - Contributes for cavitation.

River flows (River Exe, Exeter)

Urban drainage elements (U.Sheffield)

Stepped spillways (FH-Aachen)

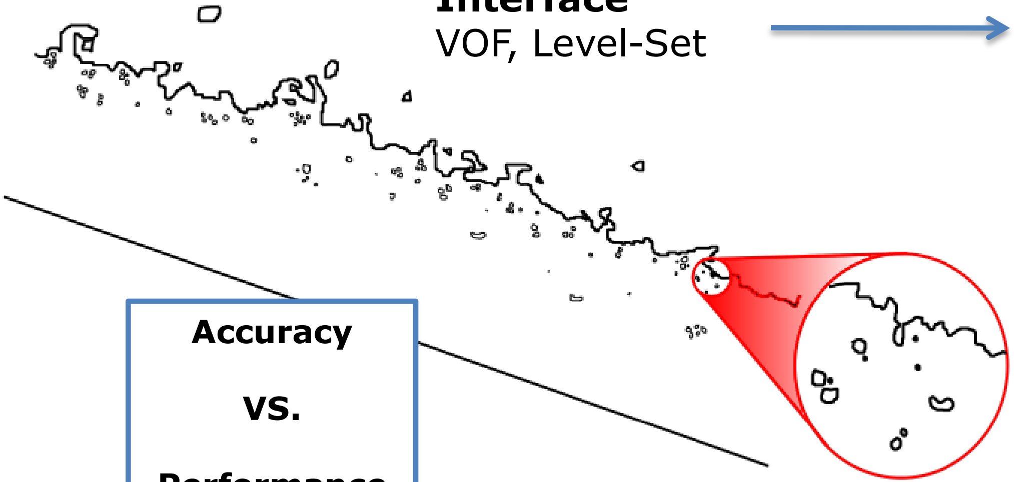


Plunging jets

Video from Kiger and Duncan 2012
$V_0$=4.4m/s ; $d_0$=8.5mm ; h=12mm

# Air-entrainment modelling

**Interface**
VOF, Level-Set



**Accuracy**

**vs.**

**Performance**

**Multi-scale aproach**

VOF + single-phase
+
Source term for air-entrainment

**Bubbles 100-1000x smaller**
Eulerian method, Single-phase

# Mathematical formulation
## Air-entrainment model

$$\nabla \cdot \mathbf{u} = 0$$

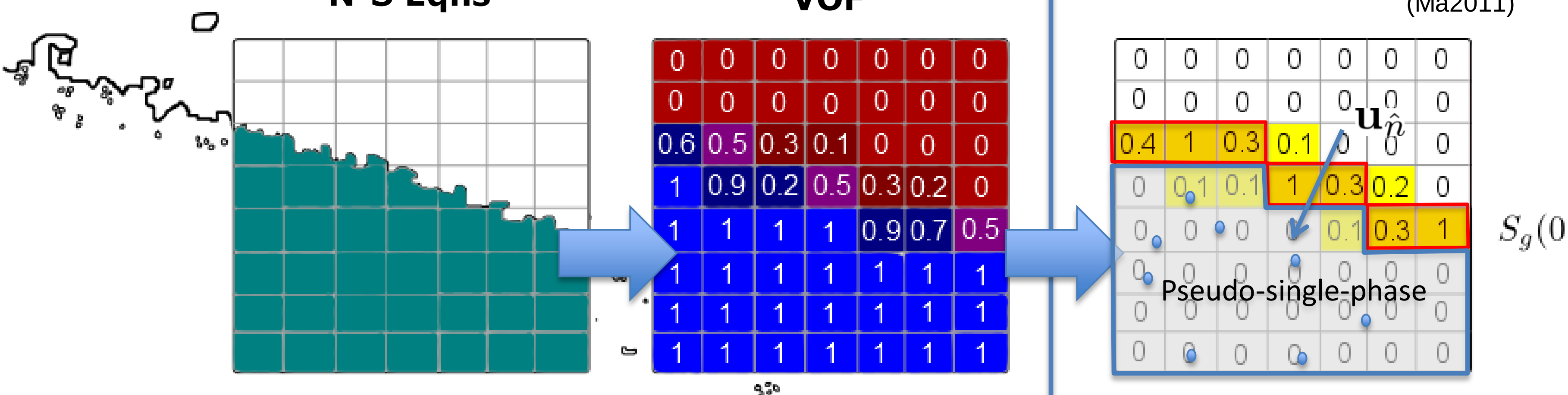
$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla P + \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{g} + \mathbf{F}$$

**Interface detection**

**Bubble creation term on interface (bubbles/m³s)**

$$S_g(0) = \frac{a}{\phi_{ent}} \left\langle \frac{\partial \bar{\mathbf{u}}_{\mathbf{n}}}{\partial \mathbf{n}} \right\rangle \delta_{fs}$$

(Ma2011)

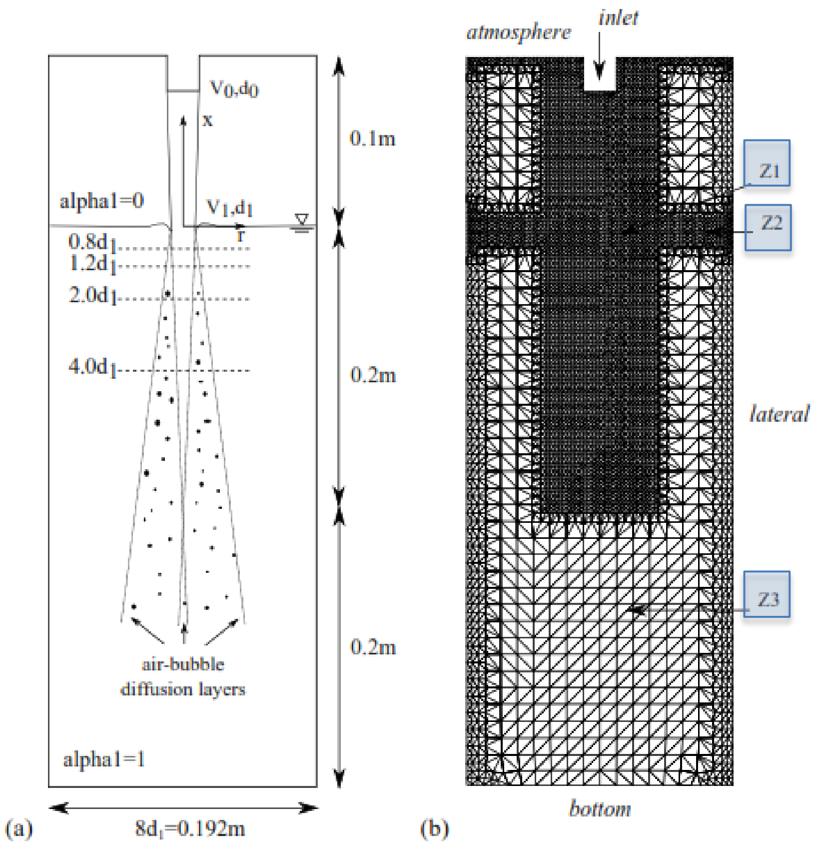


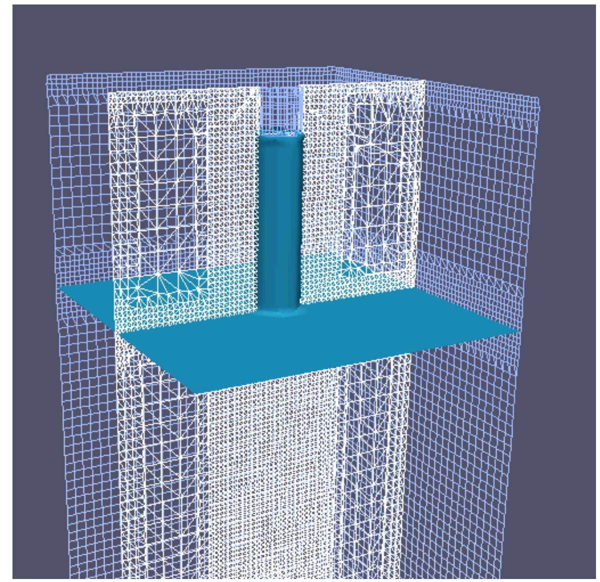

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) + \nabla \cdot [\mathbf{u}_r \alpha (1-\alpha)] = 0$$

$$a_3 \approx \frac{u'^2}{2g} = C_a \frac{k}{g}$$

$U_0$=3.5m/s ; $d_0$=24mm ; $Re_0$=5.5x$10^7$

# 3D plunging jet

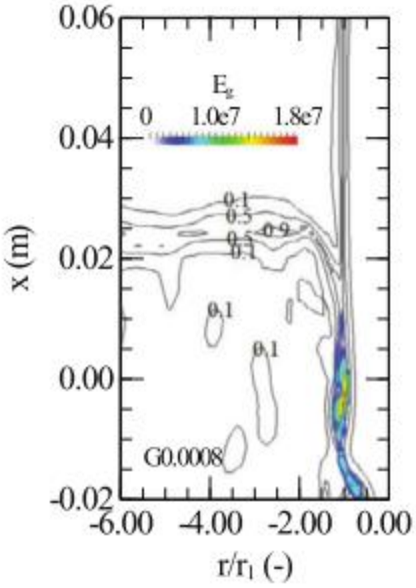


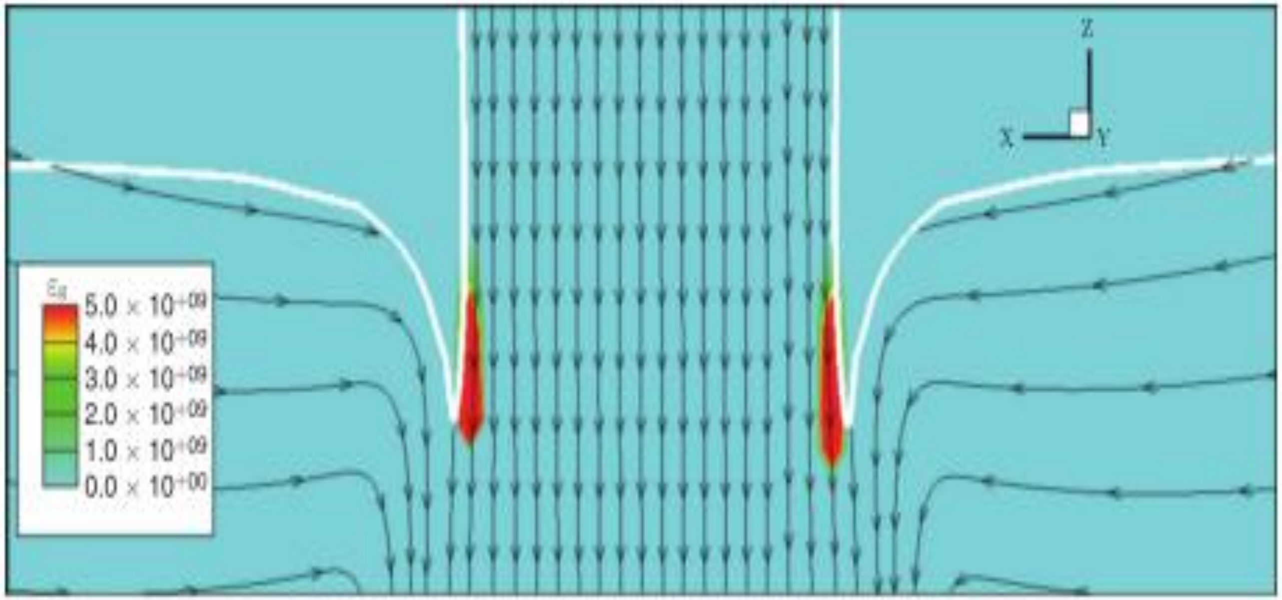



| | Refinement zones | | |
|---|---|---|---|
| | Z1 | Z2 | Z3 |
| G0.005 | 0.005 | 0.005 | 0.005 |
| G0.0025 | 0.003 | 0.0025 | 0.005 |
| G0.00125 | 0.003 | 0.0025 | 0.005 |
| G0.0008 | 0.0008 | 0.0025 | 0.005 |

Lopes, P., Tabor, G., Carvalho, R. F., and Leandro, J. (2016). “Explicit calculation of natural aeration using a Volume-of-Fluid model.” *Applied Mathematical Modelling*.

# Production term




Ma et al. (2011)

$$S_g(0) = \frac{a}{\phi_{ent}} \left\langle \frac{\partial \bar{\mathbf{u}}_\mathbf{n}}{\partial \mathbf{n}} \right\rangle \delta_{fs} \qquad a_3 \approx \frac{u'^2}{2g} = C_a \frac{k}{g}$$

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# Surface disturbances

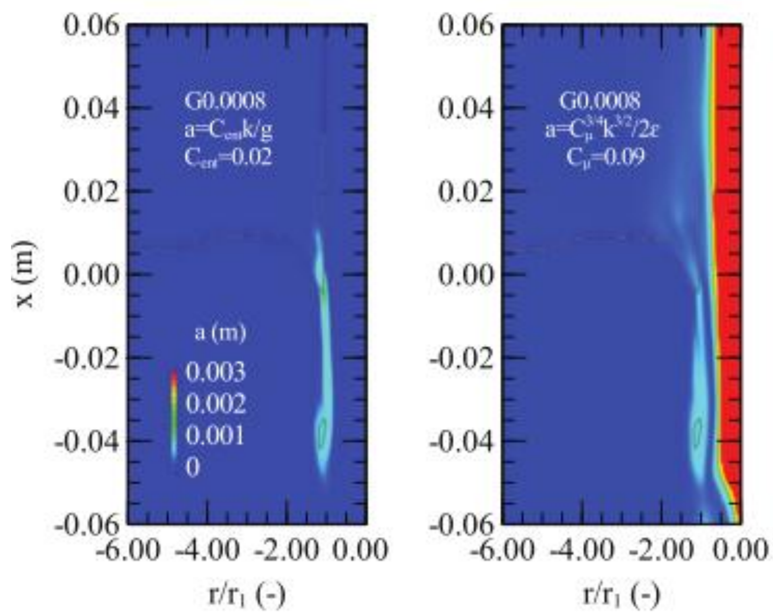
$$S_g(0) = \frac{a}{\phi_{ent}} \left\langle \frac{\partial \bar{\mathbf{u}}_\mathbf{n}}{\partial \mathbf{n}} \right\rangle \delta_{fs}$$

Ma et al. (2011)

$$a_3 \approx \frac{u'^2}{2g} = C_a \frac{k}{g}$$

Lopes et al. (2016)

$$a_1 = C_\mu^{3/4} \frac{k^{3/2}}{2\varepsilon}$$




Lopes, P., Tabor, G., Carvalho, R. F., and Leandro, J. (2016). "Explicit calculation of natural aeration using a Volume-of-Fluid model." *Applied Mathematical Modelling*.

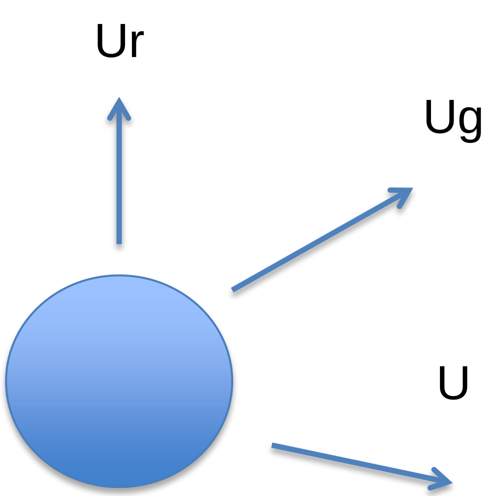
# Slip model

## Continuity equation of bubbly phase

$$\frac{\partial \alpha_g}{\partial t} + \boldsymbol{\nabla} \cdot (\bar{\mathbf{u}}_g \alpha_g) - \nabla \cdot (\Gamma_{\alpha_g} \nabla \alpha_g) = S_g$$

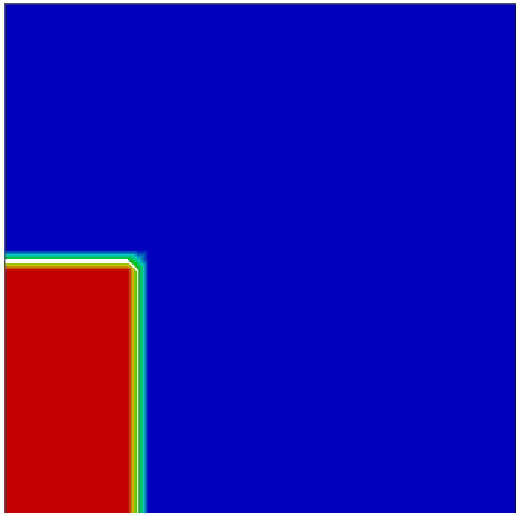
$$\bar{\mathbf{u}}_g = \bar{\mathbf{u}}_l + \bar{\mathbf{u}}_r$$

+ changes in density mixture

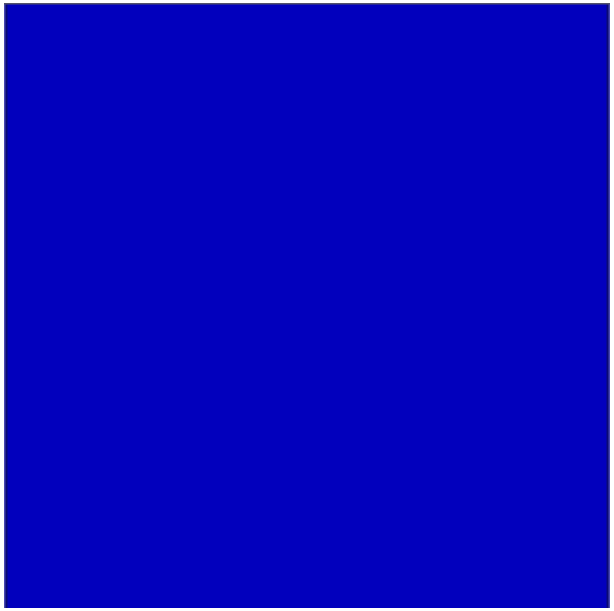


# Dambreak w/o obstacle

alpha.water

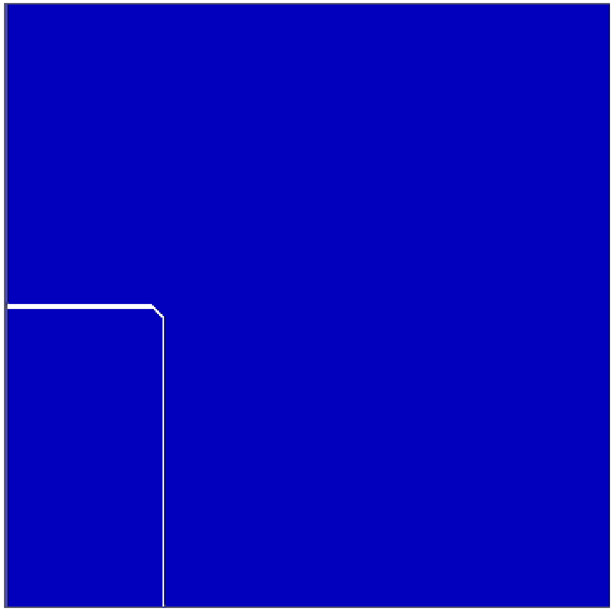


alphag



Ug = U

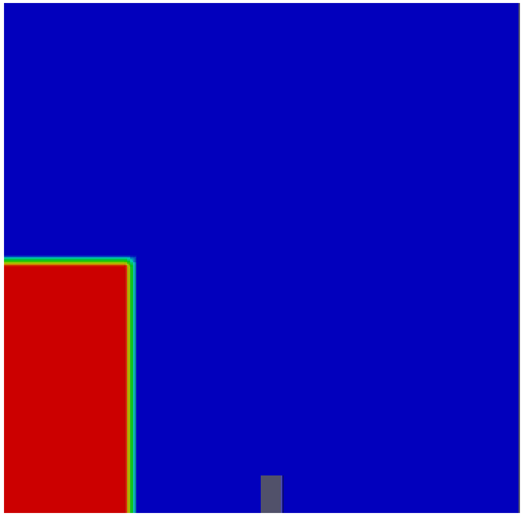
alphag



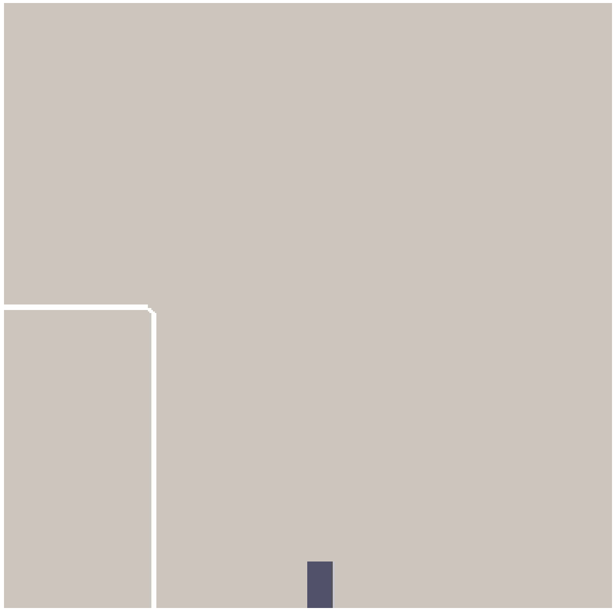
Ug = U + Uslip

# Dambreak w/ obstacle

alpha.water



alphag



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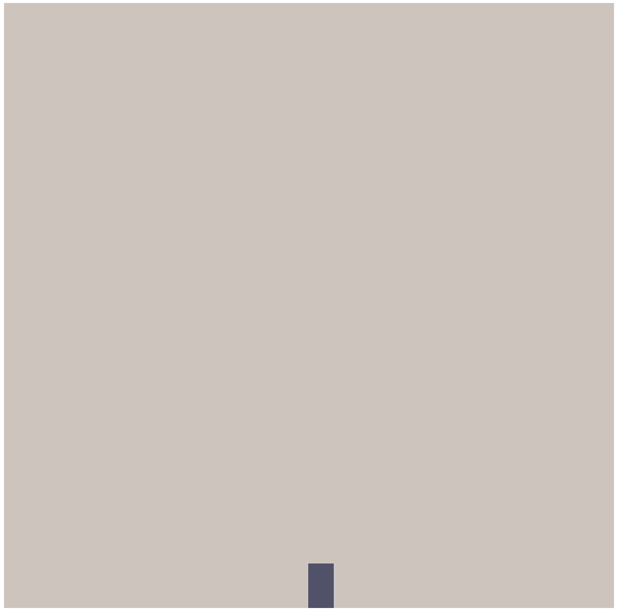

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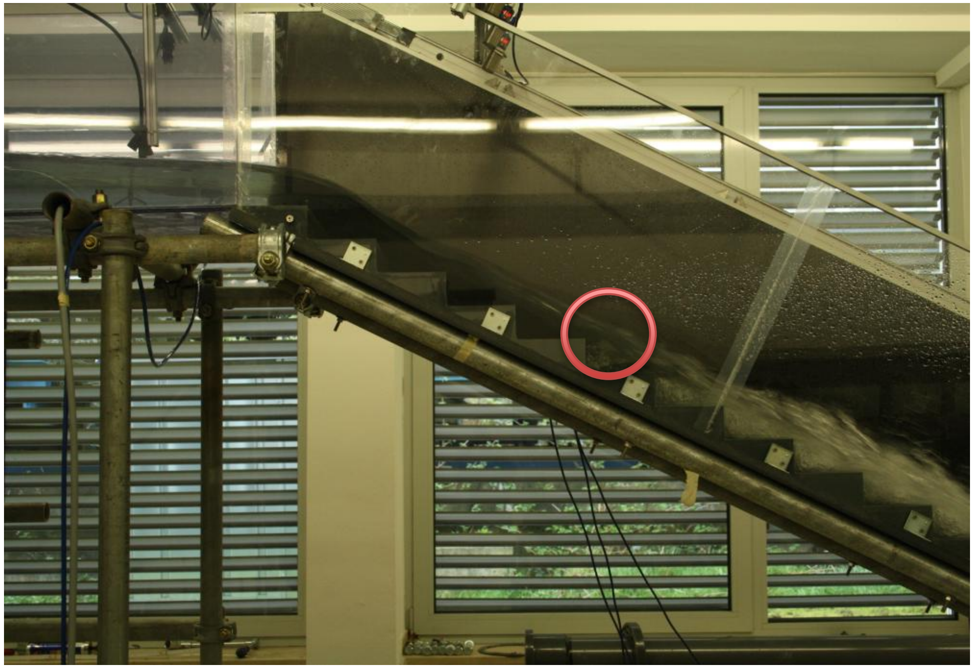
# Stepped Spillway

## Aeration at free-surface

FH-Aachen, Germany

28 steps (0.12x0.06x0.5m)
Slope = 26.6º
$q=0.07\ m^3/s/m$
$h_c=0.08$ and $Fr=4.27$

Entrainment at 5th step



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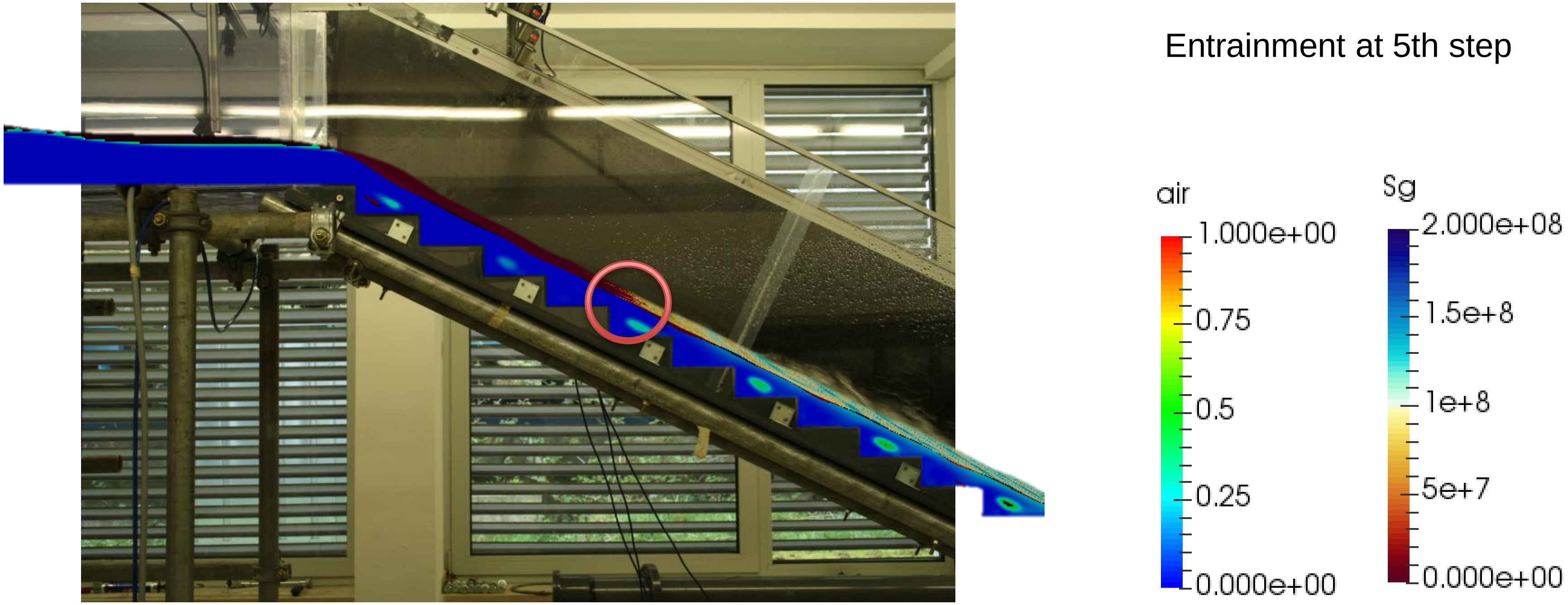
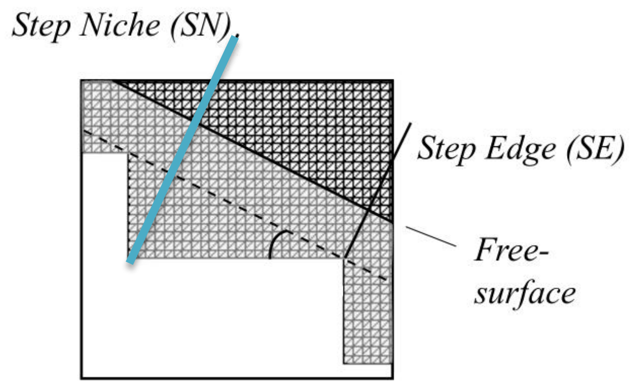
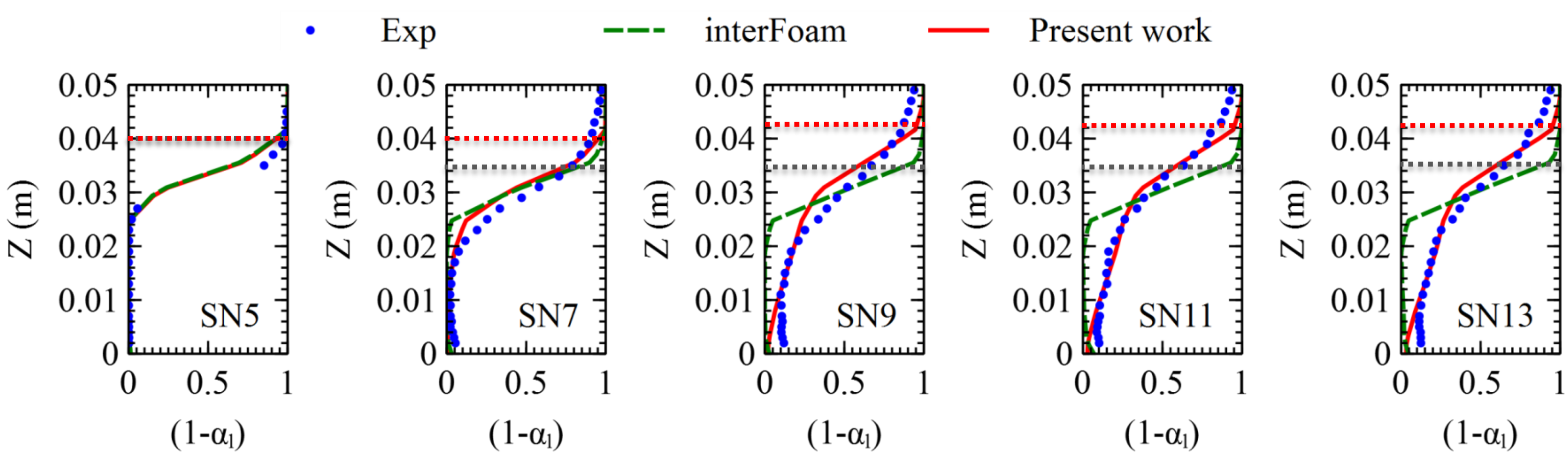


# Stepped Spillway

## Air-concentration profiles



# Conclusions

- The model can be used to **detect** the air-entrainment position;
- Slip velocity model was used to simply calculate the **advection** of the dispersed phase;
- **Air-concentration profiles** were in accordance to the experiments in a stepped spillway structure.

Thank You.

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