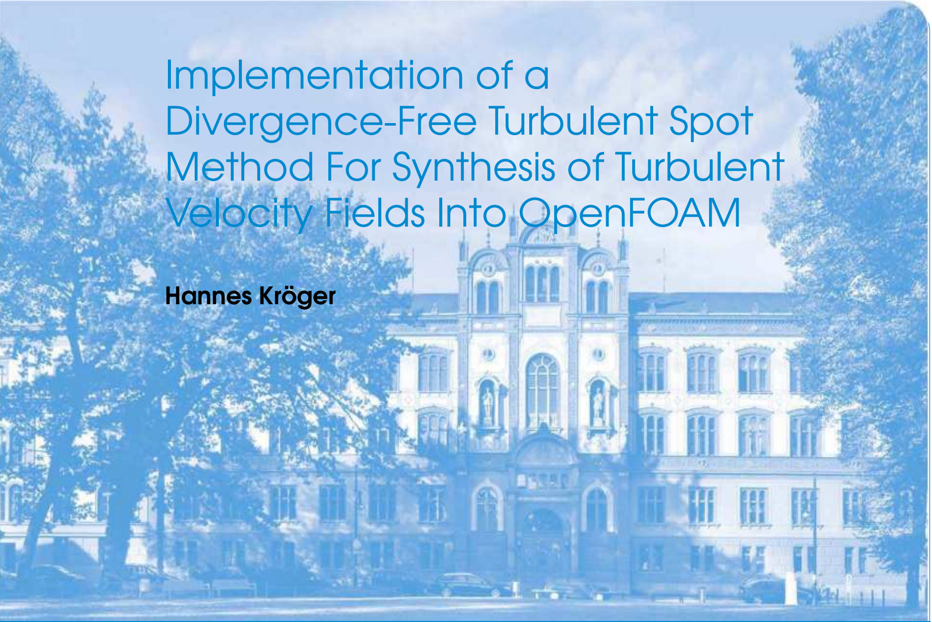


Implementation of a Divergence-Free Turbulent Spot Method For Synthesis of Turbulent Velocity Fields Into OpenFOAM

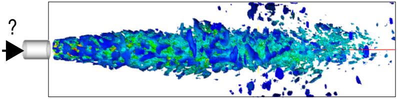
Hannes Kröger

Contents

Motivation

- For a detailed analysis of unsteady flows, a partial (⇒DES[1], LES[2]) or full (⇒DNS[3]) resolution of the turbulent motions is necessary
- DES and LES find already widespread application in industrial practice
- There is often already a turbulent flow at the inflow boundaries. A spatially and temporally resolved turbulent velocity field has to be prescribed there: [4]

$$\vec{U}(\vec{x}, t) = \langle \vec{U} \rangle (\vec{x}) + \vec{u}(\vec{x}, t)$$

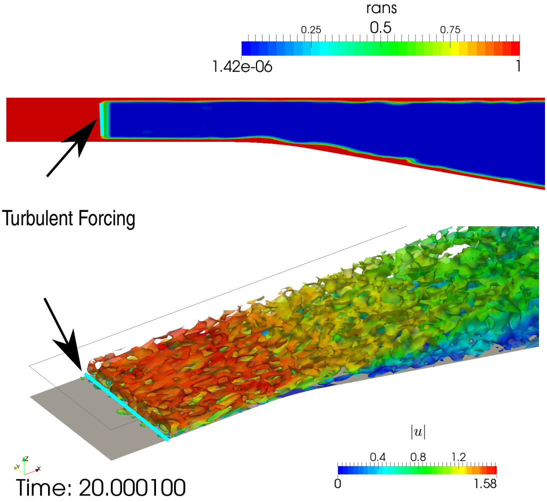


⇒The velocity fluctuations have to be generated somehow

[1] Detached-Eddy Simulation
[2] Large-Eddy Simulation
[3] Direct Numerical Simulation
[4] temporal average $\langle \phi \rangle = \int_{-\infty}^{\infty} \phi dt$

Motivation II

- DES: Turbulence insertion triggers RANS-to-LES transition
- Synthesis of realistic turbulent structures is required



Requirements

$\vec{U}(\vec{x}, t) = \langle \vec{U} \rangle(\vec{x}) + \vec{u}(\vec{x}, t)$

In order to act as turbulence, the generated velocity fluctuations need to fulfill a number of properties:

1. Reynolds Stresses, Amplitude: $\langle u_i u_j \rangle(\vec{x})$
2. Spatial Correlation, i.e. spectrum: $R_{ij}(\vec{x}, \vec{\eta}) = \frac{\langle u_i(\vec{x},t) u_j(\vec{x}+\vec{\eta},t) \rangle}{\langle u_i(\vec{x},t) u_j(\vec{x},t) \rangle}$
3. out of it, the length scales follow: $L_{ij}(\vec{x}, \vec{e}_\eta) = \int_0^\infty R_{ij}(\vec{x}, \eta \vec{e}_\eta) d\eta$
4. Continuity constraint $\nabla \cdot \vec{u} = 0$

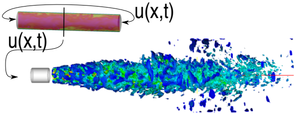
   $\Rightarrow$violation may lead to numerical problems and/or parasitic pressure fluctuations

Possible approaches for artificial generation of turbulent fluctuations:

1. Random Velocity Fluctuations
in every discrete point with specified amplitude.

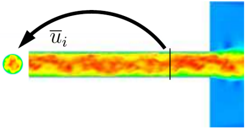
- Fluctuations are spatially and temporally uncorrelated $\Rightarrow L_{ij} = 0$
- Divergence-free constraint is violated, strong damping of fluctuations downstream $\Rightarrow$Unfeasible

2. Precursor-Simulation
Generation of turbulent fluctuations by auxiliary simulation with periodic boundary conditions.



- Computationally expensive
- Restricted to simple, generic flows

3. Recycling-Method
Extension of domain upstream and extraction of turbulent velocities from the interior domain.



- Increase of computational expense
- Only applicable to fully developed, stationary flows (tube or channel flow)

4. Synthetic Turbulence Generation

Multiple methods have been proposed:

- Sinusoidal Modes by Kraichnan
  - Superposition of global basis functions
  - Disadvantage: inhomogeneous statistics difficult
- Digital filtering method by Klein
  - Disadvantage: constant time step and grid required
  - reproduction of autocorrelation function difficult
- Superposition of “turbulent“ structures $\Rightarrow$structure based methods

The Turbulent Spot Method

- a.k.a Synthetic-Eddy-Method (SEM)
- superposition of a number of local, compact velocity fields (the ”Turbulent Spots“) on a mean velocity field.

Thereby:

- The structures are randomly distributed
- They are convected by the mean velocity through the inflow boundary
- The inner velocity distribution is scaled by a random parameter
- The size of the structures controls the length scale

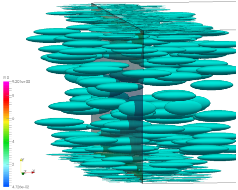


Figure: Snapshot of a population of turbulent structures around the inlet

Inner Velocity Distribution

- The inner velocity distribution $\vec{u}$ determines the autocorrelation function (= energy spectrum):

$$R_{ij}(\vec{x},\vec{\eta}) = \frac{\iiint\limits_{(V)} u_i(\vec{x})u_j(\vec{x}+\vec{\eta})dxdydz}{\iiint\limits_{(V)} u_i(\vec{x},t)u_j(\vec{x},t)dxdydz}$$

- The decay distance of the autocorrelation function determines the length scales

$$L_i(\vec{x}) = \int_0^\infty R_{ij}(\vec{x},\eta\vec{e}_i)d\eta$$

$\Rightarrow$Velocity distribution inside the spots determines energy spectrum and thus length scales

Scaling of the Fluctuations

If the algorithm produces fluctuations with unit amplitude:

1. $\langle u_{ij}^2 \rangle = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$
2. $\langle u_i \rangle = 0$

the Reynolds Stresses can be conditioned by a Cholesky transform::

$$\vec{U} = \langle \vec{U} \rangle + \mathbf{A}\vec{u}$$

with the matrix

$$\mathbf{A} = \begin{pmatrix} \sqrt{R_{11}} & 0 & 0 \\ R_{21}/A_{11} & \sqrt{R_{22} - A_{21}^2} & 0 \\ R_{31}/A_{11} & (R_{32} - A_{21}A_{31})/A_{22} & \sqrt{R_{33} - A_{31}^2 - A_{32}^2} \end{pmatrix}$$

⇒But: Cholesky transform deteriorates second-order statistics and continuity properties of generated turbulence

Current Activities

- The Structure Based Method is developed by LEMOS since 2003
- Realized features so far:
  - Prescribed inhomogeneous and anisotropic Reynolds Stresses
  - Prescribed energy spectrum and length scale (inhomogenenous, but isotropic $L_x = L_y = L_z$)
  - Not strict divergence-free [5]
- Developments in the current project:
  - Prescribed length scale (anisotropic)
  - Strictly divergence-free
  - Extensive validation

[5] because of Cholesky transformation

Implementation

- The Structure Based Method was implemented in OpenFOAM ("InflowGenerator BC")
- Tasks in this context:
  - scaling the fluctuation amplitudes to fulfill Reynolds Stresses
  - placement of the spots

Thereby:

- Produce structures online, no preprocessing steps
- Avoid numerical calibration of statistics
- Prescribe as a spatial field (inhomogeneous!):
  - length scale
  - the Reynolds Stresses

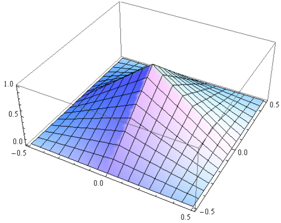
Types of Turbulent Structures

1.Spots

Direct ansatz functions for velocity

Hat Spot

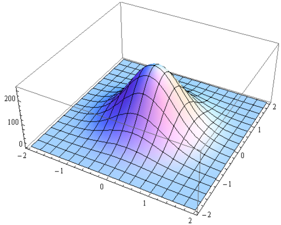
- velocity distribution: tent function



- Simple
- RS scaling by Cholesky transform
- Violates continuity constraint
- Unphysical energy spectrum

Gaussian Spot

- velocity distribution: gaussian function



- RS scaling by Cholesky transform
- Violates continuity constraint
- Energy spectrum of decaying turbulence

Types of Turbulent Structures

2. Vortons

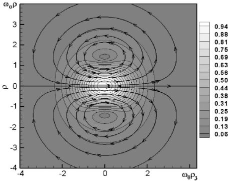
Velocity distribution derived from vector potential $\nabla \cdot \vec{u} = \nabla \cdot (\nabla \times A) = 0$

Isotropic Vorton

- symmetric velocity distribution

  $u_z = u_r = 0$

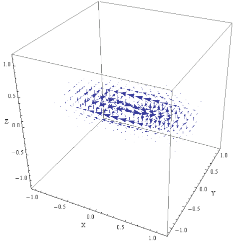
  $u_\theta = \pi \sqrt{\frac{2C_E}{C_1}} k_0^5 \exp(-k_0^2 r^2/2) r$



- RS scaling by Cholesky transform
- Fulfills continuity constraint only if RS are isotropic
- Energy spectrum of decaying turbulence

New: Anisotropic Vorton

- Most recent development
- unsymmetric velocity distribution from transformed vector potential



- No Cholesky transform required
- Fulfills continuity constraint always
- Unphysical energy spectrum

Anisotropic Vorton

- The internal velocity field follows from the vector potential $\vec{A}$:

$$\vec{u} = \nabla \times \vec{A}$$

For isotropic turbulence, a symmetric vector potential $\vec{A} = A(r)\vec{e}_z$ was derived earlier (Kornev 2007[6]).

- To account for anisotropy, scaling parameters are introduced into the vector potential:

$$\vec{A} = \exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2}\right)\right]\begin{pmatrix} x\gamma_x \\ y\gamma_y \\ z\gamma_z \end{pmatrix}$$

- with the free parameters $\gamma_x, \gamma_y, \gamma_z, \sigma_x, \sigma_y$ and σ_z.

[6] Nikolai Kornev and Egon Hassel. “Synthesis of homogeneous anisotropic divergence-free turbulent fields with prescribed second-order statistics by vortex dipoles”. In: *Physics of Fluids* 19.5 (2007).

Anisotropic Vorton

- Closed expressions for the Reynolds stresses can be derived:

$$\mathbf{R} = \frac{\pi^{3/2}}{4} \begin{pmatrix} \frac{\sigma_x(\gamma_y\sigma_y^2 - \gamma_z\sigma_z^2)^2}{\sigma_y\sigma_z} & 0 & 0 \\ 0 & \frac{\sigma_y(\gamma_x\sigma_x^2 - \gamma_z\sigma_z^2)^2}{\sigma_x\sigma_z} & 0 \\ 0 & 0 & \frac{\sigma_z(\gamma_x\sigma_x^2 - \gamma_y\sigma_y^2)^2}{\sigma_x\sigma_y} \end{pmatrix}$$

- also for the length scales

$$L_i = \int_0^\infty r_{ii}(\eta\vec{e}_i)\mathrm{d}\eta = \sqrt{\pi}\sigma_i$$

for $i = x, y, z$.

$\Rightarrow$From prescribed Reynolds stresses and length scales, the free parameters can be deduced.

Anisotropic Vorton

- But: not all combinations of length scales can be reproduced. A solution requires:

$$\pm\sqrt{R_2\frac{L_1 L_3}{L_2}} - (\pm\sqrt{R_3\frac{L_1 L_2}{L_3}}) = \pm\sqrt{R_1\frac{L_2 L_3}{L_1}}$$

or

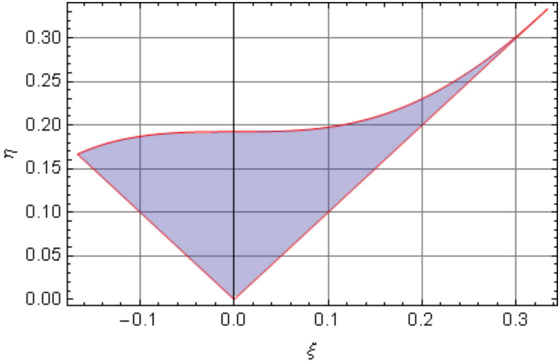
$$L_3 = \frac{\pm L_1 L_2\sqrt{R_3}}{\pm L_2\sqrt{R_1} + \pm L_1\sqrt{R_2}}$$

$\Rightarrow$Prescription of two length scales determines the third.

- Especially in the case of isotropic turbulence $R_1 = R_2 = R_3 = R$ with $L_1 = L_2 = L$ the condition $L_3 = L/2$ needs to be fulfilled.

Realizable Anisotropy States

Lumley-Triangle

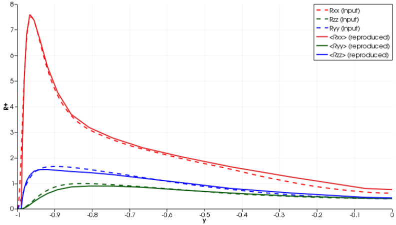


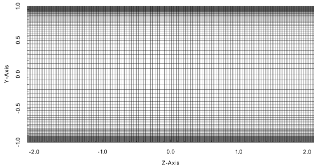
- shaded area is reproducable by ansiotropic vortons

  ⇒entire area

Reproduction of Reynolds stresses

Reproduction of Reynolds stresses in LES of channel flow at $Re_\tau = 395$.





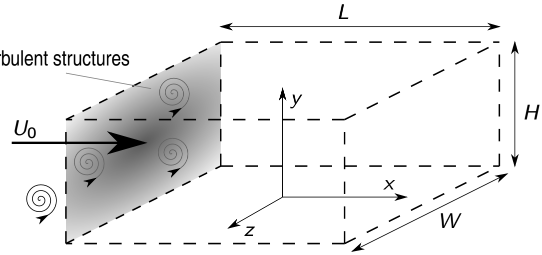
- Dashed: prescribed input profiles
- Solid: reproduced profiles

Test Cases

Test cases

- spatially Decaying Turbulence
- channel flow

Spatially Decaying Turbulence

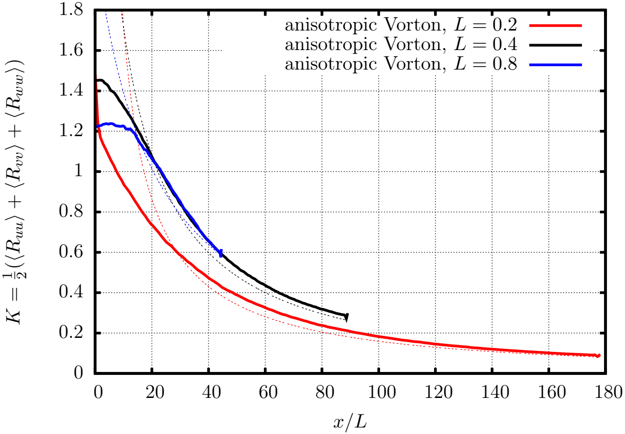


CFD domain for spatially decaying turbulence test case

Parameter	Value
SGS model	dyn. Smagorinsky
Domain size	$L = 36mm$
	$H = W = 3m$
Grid	$n_x = 384$
	$n_y = n_z = 64$
Velocity	$U_0 = 20m/s$
Fluctuation	$u' = 1m/s$
Viscosity	$\nu = 3.5 \times 10^{-4} \frac{m^2}{s}$

- resembles simple grid turbulence in wind tunnel
- uniform convection speed
- isotropic turbulence at inlet

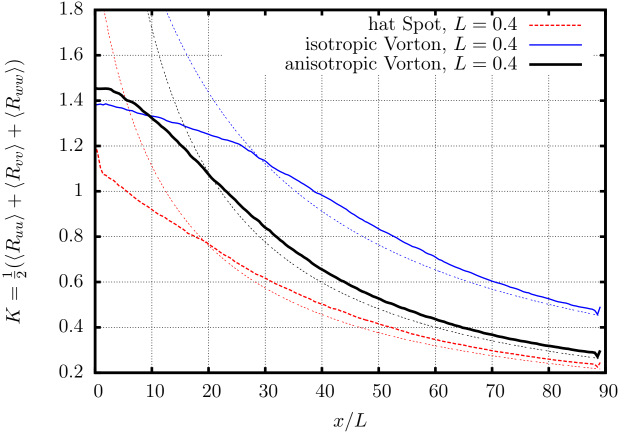
Length Scales



Spatially decaying turbulence: turbulent kinetic energy vs. distance from inlet for different prescribed integral length scales

⇒decay near inlet depends on length scale

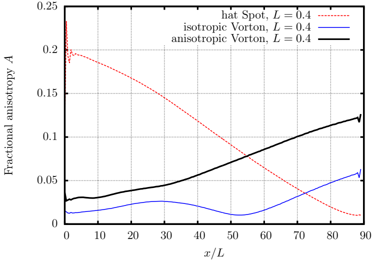
Structure Types



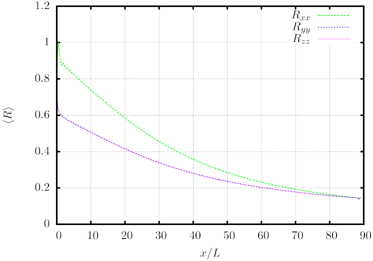
Spatially decaying turbulence: turbulent kinetic energy vs. distance from inlet for different turbulent structure types.

Anisotropy

a)



b)



Spatially decaying turbulence:

a) anisotropy of Reynolds stresses vs. distance from inlet for different turbulent structure types

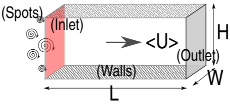
anisotropy measure: $A = \frac{\sqrt{\hat{R}^*}}{\sqrt{R_{uu}^2 + R_{vv}^2 + R_{ww}^2}}$ with

$\hat{R}^* = (R_{uu} - \hat{R})^2 + (R_{vv} - \hat{R})^2 + (R_{ww} - \hat{R})^2$ and $\hat{R} = (R_{uu} + R_{vv} + R_{ww})/3$.

b) Individual Reynolds stress components for the case hat Spot with $L = 0.4$

$\Rightarrow$ pronounced anisotropy for non-divergence free structures

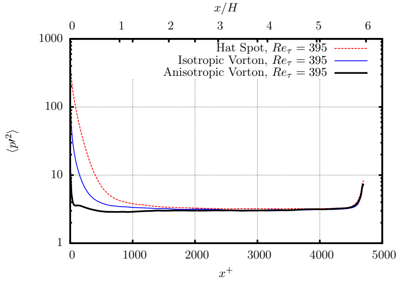
Channel flow simulations



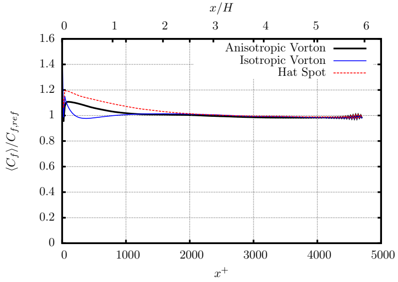
Re_τ	structure type
180	anisotropic vorton
395	hat spot isotropic vorton anisotropic vorton
590	anisotropic vorton

Reynolds number Re_τ	Domain size $L \times H(=2\delta) \times W$	Resolution $\Delta_x^+ \times \Delta_{y,wall}^+ \times \Delta_z^+$	Grid size $n_x \times n_y \times n_z$
180	$12 \times 2 \times 4.2$	$20 \times 2 \times 10$	$108 \times 48 \times 75$
395	$12 \times 2 \times 4.2$	$20 \times 2 \times 10$	$237 \times 64 \times 165$
590	$12 \times 2 \times 4.2$	$20 \times 2 \times 10$	$354 \times 75 \times 247$

Channel Flow - Adaption Length



Inflow channel simulations: comparison of pressure fluctuations vs. axial distance for different types of turbulent structures (at $Re_\tau = 395$)

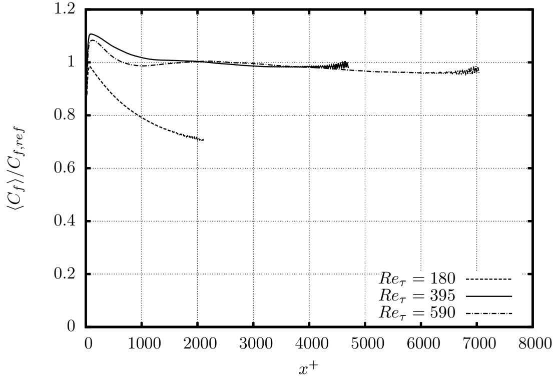


Inflow channel simulations: comparison of friction coefficient vs. axial distance for different types of turbulent structures (at $Re_\tau = 395$)

$\Rightarrow$least pressure noise for fully divergence-free formulation

$\Rightarrow$least adaption length for divergence-free formulations

Dependence on Re_τ

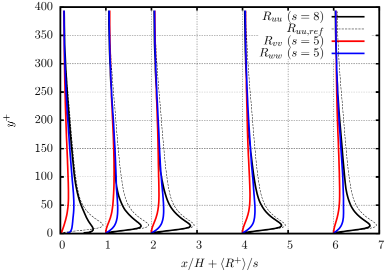


Inflow channel simulations: comparison of friction coefficient vs. axial distance for different Reynolds numbers (using the anisotropic vorton)

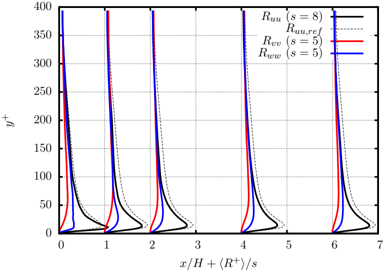
⇒unsuccessful for smallest Re_τ (turbulence is not sustained)

⇒slight reduction of adaption length with Reynolds number

Renolds Stress Profiles



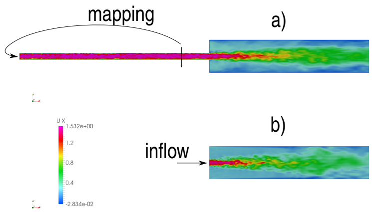
Reynolds stresses at different sections in the channel for isotropic vortons ($Re_\tau = 395$)



Reynolds stresses at different sections in the channel for anisotropic vortons ($Re_\tau = 395$)

⇒better agreement of Reynolds stresses during adaption length with anisotropic vorton formulation

Computational Efficiency

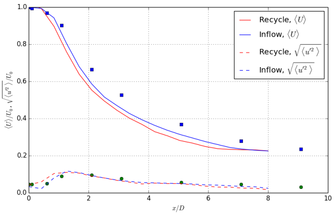


jet mixer test case:

a). using recyling method (1100k cells)

b). using inflow generator (670k cells, ≈-40%)

(Identical resolution of the domain downstream of nozzle in both cases.)



Comparison

- recycling case ≈ 49700 sec. wallclock time / sec. simulation time
- rnflow generator: ≈ 26000 sec. wallclock time / sec. simulation time

⇒≈ +91% increased wallclock time by usage of recycling method

Summary

- The Structure Based Method is a relatively simple and elegant method for synthesis of turbulent fields.
- Implementations so far were restricted to isotropic length scales and divergent velocity fields.
- The most important challenge is the combination of anisotropy with the divergence-free constraint.
- Formulation of anisotropic divergence-free vortons has been derived and implemented.

Outlook

- Work in Progress: elimination of length scale restriction

  $\Rightarrow$by superposition of uncorrelated isotropic and anisotropic vortons.

$$R_i = \rho_i^{(iso)} + \rho_i^{(aniso)}$$

$$L_i = \frac{\lambda_i^{(iso)} \rho_i^{(iso)} + \lambda_i^{(aniso)} \rho_i^{(aniso)}}{R_i}$$

- An extensive validation and comparison with other methods for turbulence generation is currently conducted.
- Extension for compressible flows and scalar fluctuations are planned.

Thank you for your attention!

Acknowledgements

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