

Recent advances in pressure-velocity coupled solver

Tessa Uroić, Vuko Vukčević, Hrvoje Jasak

Chair of Turbomachinery
Department of Energy, Power Engineering and Environment
Faculty of Mechanical Engineering and Naval Architecture, University of
Zagreb

tessa.uroic@fsb.hr

June 25, 2016

Introduction

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Introduction

- Computational fluid dynamics (CFD) tools are regularly used for complex, coupled and non-linear heat and mass transfer problems.
- With the development of modern computers and accessibility of High-Performance Computing (HPC) clusters with substantial memory resources, early compromises in the concept of simulation tools can be revisited.
- Inter-equation coupling terms dominate the system of equations and most algorithms use conventional segregated approach, i.e. solving the equations sequentially.
- New solution techniques rely on linearisation of inter-equation coupling and solution in an implicit coupled manner. The pressure-velocity coupling in Navier-Stokes equations is linear (see momentum conservation equation).
- The benefits of the coupled approach are:
 - significant reduction in time-to-solution,
 - numerical stability of the algorithm,
 - *but*, at a cost of increased memory usage.

Equation coupling

A problem with two coupled unknown scalar fields in segregated approach:

$$\mathbf{A}(y)x = a \quad (1)$$

$$\mathbf{B}(x)y = b \quad (2)$$

where \mathbf{A} and \mathbf{B} are matrices, x and y are unknowns and a and b are sources. The system is coupled: \mathbf{A} depends on present values of y and \mathbf{B} depends on present values of x .

Solving the equations together corresponds to:

$$\begin{pmatrix} \mathbf{A}(y) & 0 \\ 0 & \mathbf{B}(x) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (3)$$

but there is still no coupling. Remove explicit linear dependencies from \mathbf{A} and place it in off-diagonal position in order to achieve implicit dependence:

$$\begin{pmatrix} \mathbf{A}' & \mathbf{A}_y \\ \mathbf{B}_x & \mathbf{B}' \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (4)$$

Segregated approach

Incompressible, steady-state Navier-Stokes equations:

$$\nabla \cdot (\mathbf{u}^{new}) = 0 \quad (5)$$

$$\nabla \cdot (\mathbf{u}^{old} \mathbf{u}^{new}) - \nabla \cdot (\nu \nabla \mathbf{u}^{new}) = -\frac{1}{\rho} \nabla p^{new} \quad (6)$$

Note: the superscripts *new* and *old* denote the coupled approach where equations are solved simultaneously.

Segregated approach (SIMPLE algorithm):

- 1 Guess the pressure field.
- 2 Solve the momentum equation using the guessed pressure field.
- 3 Compute the pressure based on the predicted velocity field.
- 4 Compute conservative face flux.
- 5 Iterate!

Note: underrelaxation must be used to achieve convergence.

The coupled matrix

Consider the system:

$$\mathbf{A}\phi = b \quad (7)$$

$$\begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \dots & a_{n,n} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \quad (8)$$

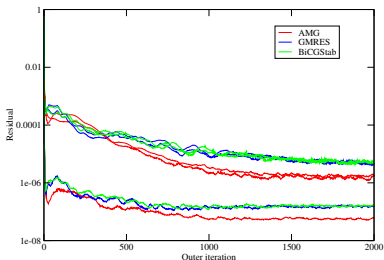
In the coupled approach, each ϕ_i is a four component vector:

$$\phi = \begin{pmatrix} u_{xi} \\ u_{yi} \\ u_{zi} \\ p_i \end{pmatrix} \quad (9)$$

Linear solvers

- The iterative linear solvers most efficient for the coupled system are the solvers based on Krylov subspace (such as some generalized formulation of the Conjugate Gradient method) and Algebraic Multigrid (AMG) solver.
- The Krylov subspace solvers (Generalised Minimised Residual - GMRES, and Biconjugate Gradient Stabilised) are suitable for general non-symmetric matrices and converge very well combined with preconditioning.
- AMG methods had some trouble converging with the older versions of the coupled solver as the matrix was ill-conditioned (eigenvalues larger than 1). Most recent test cases have shown better convergence of the AMG solver compared to the Krylov subspace solvers.

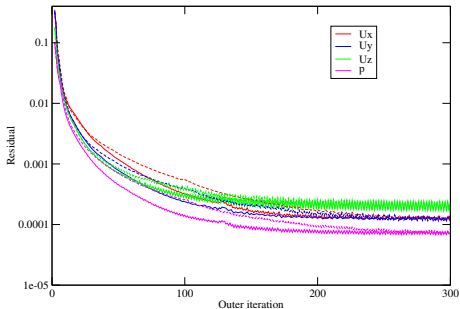
Figure 1 : Comparison of linear solvers for NACA blade case



ILUCp preconditioner

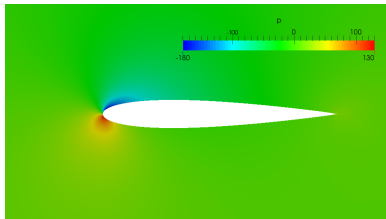
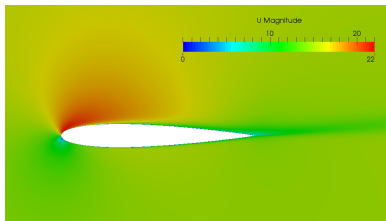
- Due to the bad eigenspectrum of the matrix, a more general preconditioner had to be implemented.
- The chosen preconditioner is incomplete LU factorization with arbitrary level of fill in.

Figure 2 : Performance of ILUCp and Gauss Seidel preconditioners with AMG



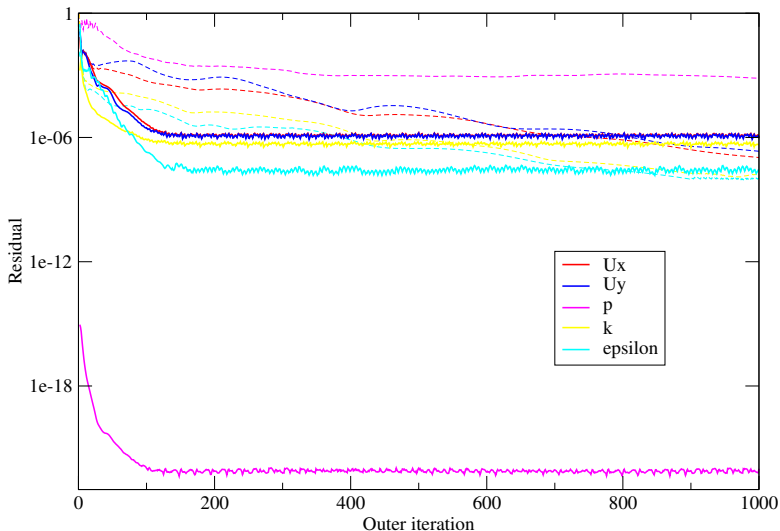
NACA blade

Figure 3 : Velocity and pressure field



NACA blade

Figure 4 : Coupled = continuous line, Segregated = dashed line



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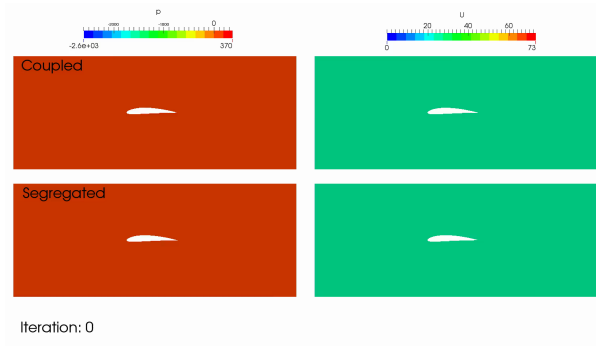
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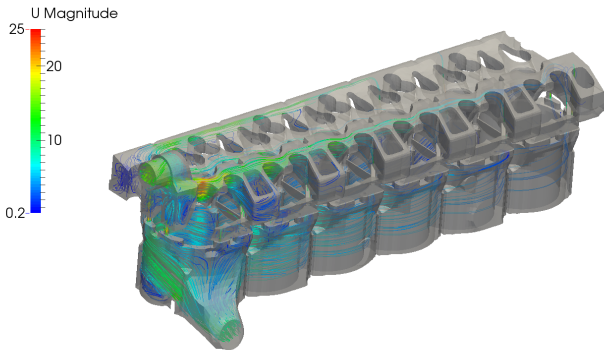
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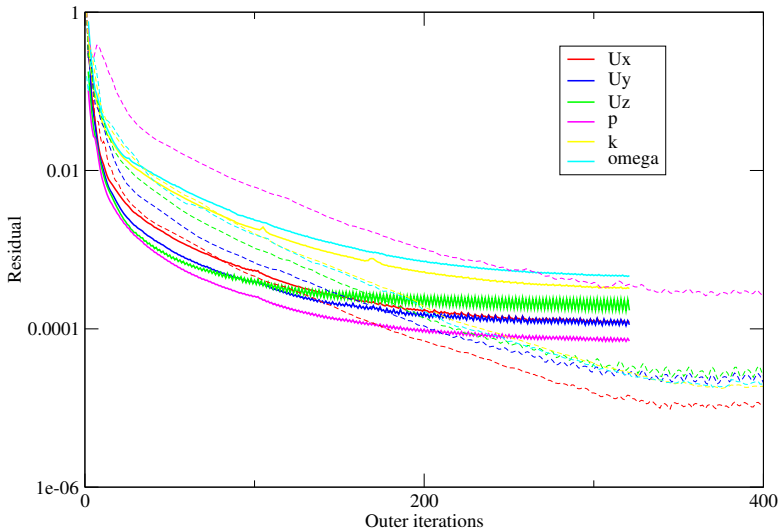
Engine cooling

Figure 5 : Engine cooling



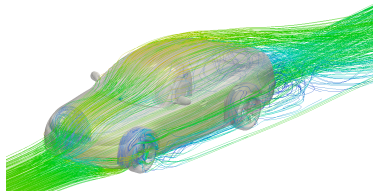
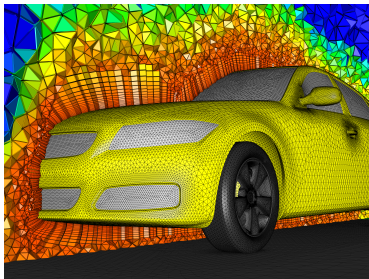
Engine cooling

Figure 6 : Coupled = continuous line, Segregated = dashed line



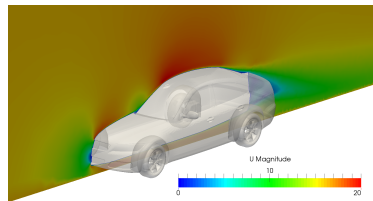
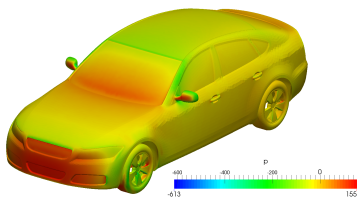
Generic car

Figure 7 : Unstructured mesh with structured boundary layer (courtesy of Pointwise) and streamlines around the car



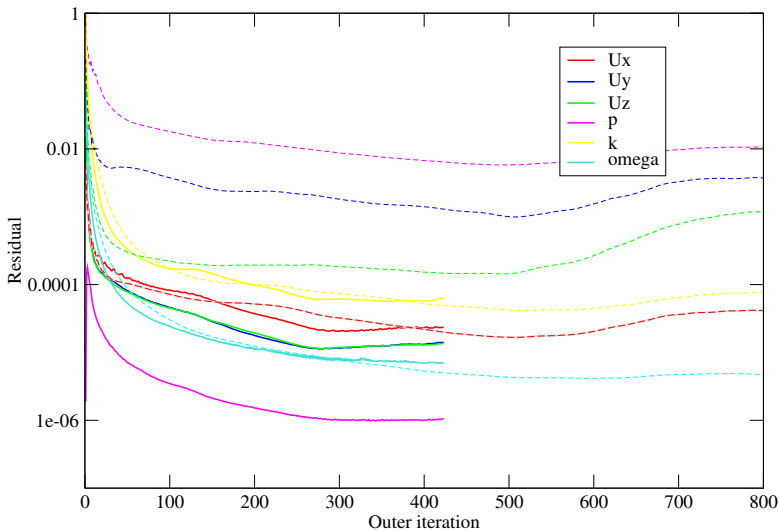
Generic car

Figure 8 : Pressure and velocity field



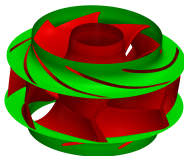
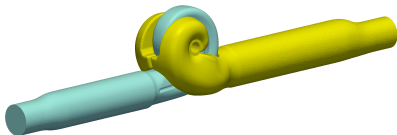
Generic car

Figure 9 : Coupled = continuous line, Segregated = dashed line



Centrifugal pump

Figure 10 : Pump casing and impeller



Centrifugal pump

Figure 11 : Pressure and turbulence kinematic energy

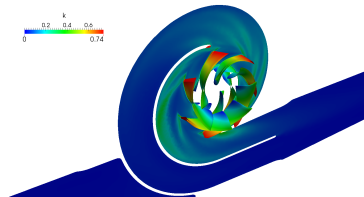
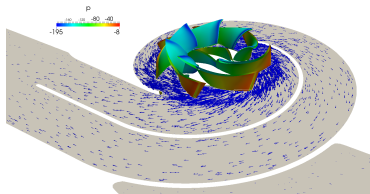
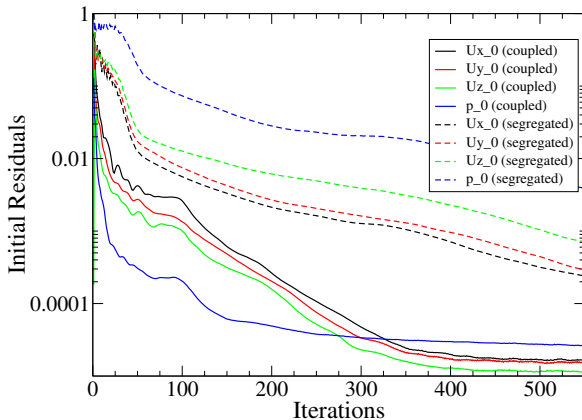


Figure 12 : Residuals



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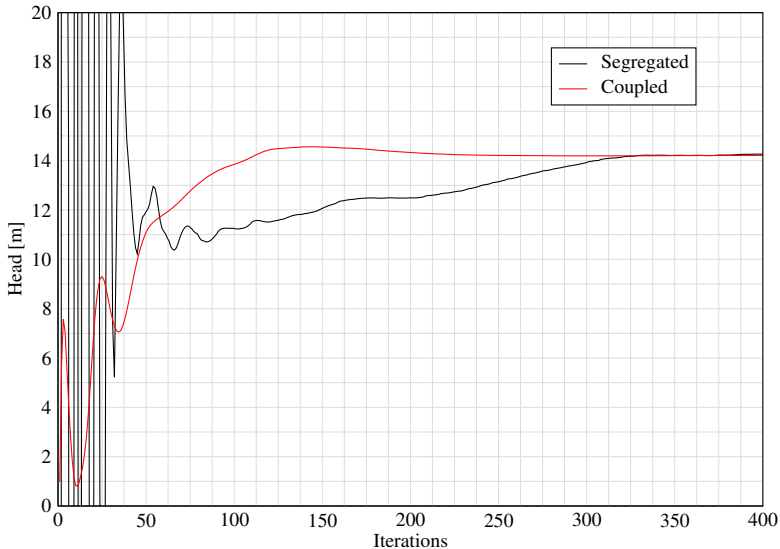
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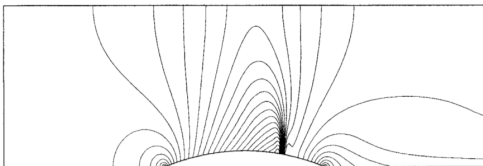
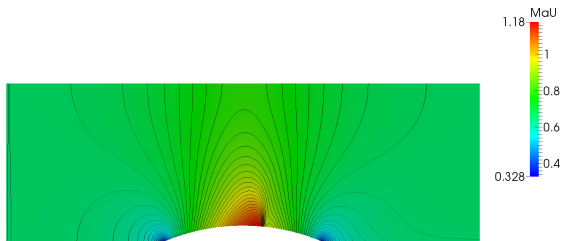
Centrifugal pump

Figure 13 : Head convergence



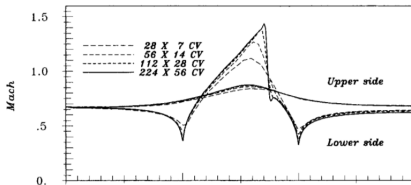
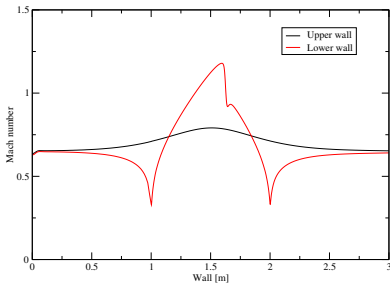
Transonic bump

Figure 14 : Mach contours - comparison to literature



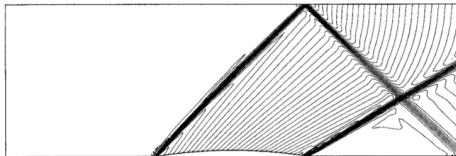
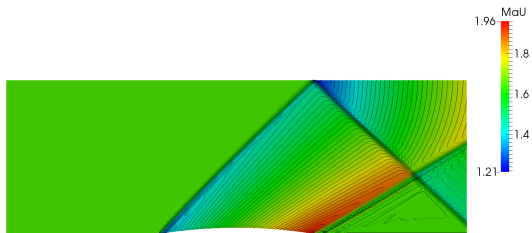
Transonic bump

Figure 15 : Mach number on walls - comparison to literature



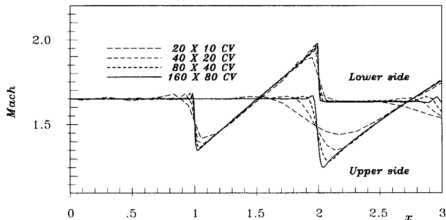
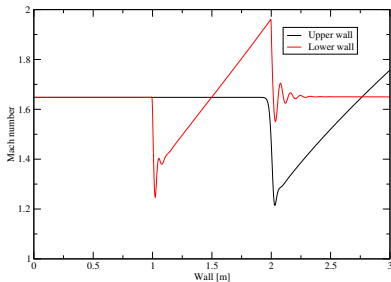
Supersonic bump

Figure 16 : Mach contours - comparison to literature



Supersonic bump

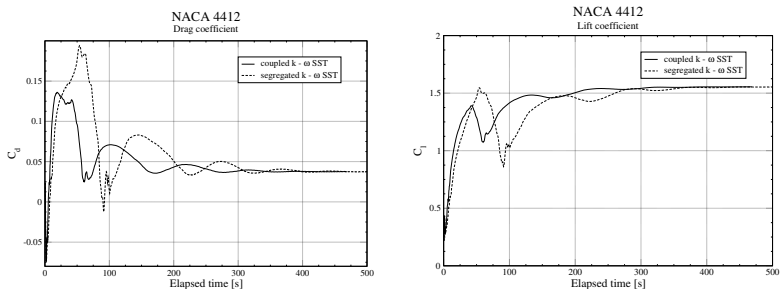
Figure 17 : Mach number on walls - comparison to literature



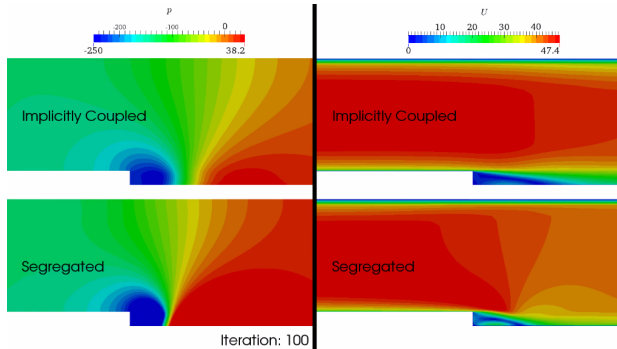
Implicitly coupled turbulence models

- Segregated treatment of turbulence models limits the coupled algorithm from reaching its full potential.
- In order to implement the implicitly coupled turbulence models in the block-matrix framework, non-linear source terms must be linearised.
- The coupled turbulence models (k - ϵ and k - ω SST) were validated and benchmarked against the segregated models. The coupled models provide faster and more stable convergence and preserve boundedness of the variables.

Figure 18 : Convergence of force coefficients for NACA blade



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Future work

The current and future work includes:

- implementation of the selective algebraic multigrid algorithm for the block matrix - **In progress**
- implementation of new preconditioners for the coupled matrix - blocked algorithms
- coupling of the mean flow equations to the turbulence model equations?

11th
OpenFOAM
Workshop

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Thank you for your attention.
Questions, please?