Harmonic Balance Method for Turbomachinery Applications

11$^{\mathrm{th}}$ OpenFOAM® Workshop

Gregor Cvijetić and Hrvoje Jasak

Faculty of Mechanical Engineering and Naval Architecture

gregor.cvijetic@fsb.hr

June 26 - 30, 2016

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Introduction

Many modern CFD problems involve temporally periodic flows:

- wave–like phenomena, flows induced by periodic boundary condition, flows with periodically moving objects
- wing oscillations, moving valves, flutter, etc.

Some of them are quite complex - turbomachinery.

Common problems:

- different specialised steady–state methods available, but usually transient simulation is needed for resolving transient effects
- high CPU times for whole simulation
- in order to neglect simulation start unsteadiness, a number of periods have to be run - one period is not enough
- in specific cases **up to 100 periods** were needed to reach periodic steady state

Harmonic Balance Method

- method for temporally periodic problems where the fundamental frequency is known in advance
- it is assumed that each primitive variable can be accurately represented by a Fourier series **in time**, using first n harmonics and mean value
- instead of running transient simulation, $2n + 1$ steady state problems for $2n + 1$ equally spaced time steps within one period are solved

- time derivative term is transformed into source terms denoting coupling with other time steps
- method is computationally more efficient and faster than conventional transient methods (substituting transient simulation with a number of steady state problems)
- memory increase
- implemented in foam-extend

Harmonic Balance for Scalar Transport

The derivation

Starting point is a scalar transport equation in condensed form:

$$\frac{\partial \mathcal{Q}}{\partial t} + \mathcal{R} = 0.$$

$\mathcal{R}$ stands for the convection and diffusion transport and source/sink terms of the transport equation:

$$\mathcal{R} = \nabla \bullet (\mathbf{u}\mathcal{Q}) - \nabla \bullet (\gamma \nabla \mathcal{Q}) - q_v$$

Discrete Fourier series expansion of scalar variable $\mathcal{Q}$ with n harmonics:

$$\mathcal{Q}(t) = \mathrm{Q}_0 + \mathrm{Q}_{\mathrm{S}_1}\sin(\omega t) + \mathrm{Q}_{\mathrm{C}_1}\cos(\omega t) + \mathrm{Q}_{\mathrm{S}_2}\sin(2\omega t) + \mathrm{Q}_{\mathrm{C}_2}\cos(2\omega t)$$
$$+ \ldots +$$
$$\mathrm{Q}_{\mathrm{S}_n}\sin(n\omega t) + \mathrm{Q}_{\mathrm{C}_n}\cos(n\omega t),$$

Expansion of $\mathcal{R}$:

$$\mathcal{R}(t) = \mathrm{R}_0 + \mathrm{R}_{\mathrm{S}_1}\sin(\omega t) + \mathrm{R}_{\mathrm{C}_1}\cos(\omega t) + \mathrm{R}_{\mathrm{S}_2}\sin(2\omega t) + \mathrm{R}_{\mathrm{C}_2}\cos(2\omega t)$$
$$+ \ldots +$$
$$\mathrm{R}_{\mathrm{S}_n}\sin(n\omega t) + \mathrm{R}_{\mathrm{C}_n}\cos(n\omega t).$$

Grouped together, $2n+1$ equations are obtained:

$$\text{n for sine}\begin{cases} -\omega Q_{C_1} + R_{S_1} = 0 \\ -2\omega Q_{C_2} + R_{S_2} = 0 \\ \dots \\ -n\omega Q_{C_n} + R_{S_n} = 0 \end{cases}$$

$$\text{mean} \qquad R_0 = 0$$

$$\text{n for cosine}\begin{cases} \omega Q_{S_1} + R_{C_1} = 0 \\ 2\omega Q_{S_2} + R_{C_2} = 0 \\ \dots \\ n\omega Q_{S_n} + R_{C_n} = 0, \end{cases}$$

Written in matrix form:

$$\omega \mathbf{A}\mathbf{Q} + \mathbf{R} = 0.$$

Matrix **A**, **Q** and **R** are coefficients matrices:

$$\mathbf{A} = \begin{bmatrix}
 & & & & & 0 & -1 & & & & \\
 & & & & & 0 & & -2 & & & \\
 & & 0 & & & . & & & -3 & & \\
 & & & & & & & & & \ddots & \\
 & & & & & . & & & & & \\
 & & & & & . & & & & & -n \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
1 & & & & & . & & & & & \\
 & 2 & & & & . & & & & & \\
 & & 3 & & & . & & & 0 & & \\
 & & & \ddots & & 0 & & & & & \\
 & & & & n & 0 & & & & &
\end{bmatrix}, \quad
\mathbf{Q} = \begin{bmatrix} Q_{S_1} \\ Q_{S_2} \\ Q_{S_3} \\ \vdots \\ Q_{S_n} \\ Q_0 \\ Q_{C_1} \\ Q_{C_2} \\ Q_{C_3} \\ \vdots \\ Q_{C_n} \end{bmatrix}, \quad
\mathbf{R} = \begin{bmatrix} R_{S_1} \\ R_{S_2} \\ R_{S_3} \\ \vdots \\ R_{Sn} \\ R_0 \\ R_{C_1} \\ R_{C_2} \\ R_{C_3} \\ \vdots \\ R_{C_n} \end{bmatrix}.$$

Using a DFT transformation matrix E:

$$\mathbf{Q} = \mathbf{E}\mathcal{Q}(t) \qquad \text{and} \qquad \mathbf{R} = \mathbf{E}\mathcal{R}(t),$$

it is possible to keep the equation in frequency domain but use variables in the **time domain**:

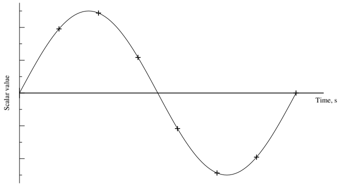
$$\omega \mathbf{A}\mathbf{E}\mathcal{Q} + \mathbf{E}\mathcal{R} = 0$$

Multiplying the equation by transformation matrix $\mathbf{E}^{-1}$ from the left, the final form of the Harmonic Balance transport equation in **time domain** is obtained!

ORIGINAL EQUATION	**HB FORMULATION**
$\dfrac{\partial \mathcal{Q}}{\partial t} + \mathcal{R} = 0$	$\omega \mathbf{E}^{-1}\mathbf{A}\mathbf{E}\boldsymbol{\mathcal{Q}} + \boldsymbol{\mathcal{R}} = 0$

Time-domain matrices $\boldsymbol{\mathcal{Q}}$ and $\boldsymbol{\mathcal{R}}$ contain variable in $2n+1$ time steps:

$$\boldsymbol{\mathcal{Q}} = \begin{bmatrix} \mathrm{Q}_{t_1} \\ \mathrm{Q}_{t_2} \\ \mathrm{Q}_{t_3} \\ \vdots \\ \mathrm{Q}_{t_{2n+1}} \end{bmatrix}, \quad \boldsymbol{\mathcal{R}} = \begin{bmatrix} \mathrm{R}_{t_1} \\ \mathrm{R}_{t_2} \\ \mathrm{R}_{t_3} \\ \vdots \\ \mathrm{R}_{t_{2n+1}} \end{bmatrix},$$



where t_n stands for:

$$t_1 = \frac{T}{2n+1}, \quad t_2 = \frac{2T}{2n+1}, \quad \dots, \quad t_n = \frac{nT}{2n+1}.$$

Developing the equation $\omega \mathbf{E}^{-1}\mathbf{A}\mathbf{E}\mathcal{Q} + \mathcal{R} = 0$, $2n+1$ equations are obtained:

$$\mathrm{Q}_{t_1}: \quad \nabla\cdot(\mathbf{u}\mathrm{Q}_{t_1}) - \nabla\cdot(\gamma\nabla\mathrm{Q}_{t_1}) = -\frac{2\omega}{2n+1}\left(\mathrm{Q}_{t_2}\mathrm{P}_1 + \mathrm{Q}_{t_3}\mathrm{P}_2 + \ldots + \mathrm{Q}_{t_{2n+1}}\mathrm{P}_{2n}\right)$$

$$\mathrm{Q}_{t_2}: \quad \nabla\cdot(\mathbf{u}\mathrm{Q}_{t_2}) - \nabla\cdot(\gamma\nabla\mathrm{Q}_{t_2}) = -\frac{2\omega}{2n+1}\left(\mathrm{Q}_{t_2}\mathrm{P}_1 + \mathrm{Q}_{t_3}\mathrm{P}_2 + \ldots + \mathrm{Q}_{t_{2n+1}}\mathrm{P}_{2n}\right)$$

$$\vdots$$

$$\nabla\cdot(\mathbf{u}\mathrm{Q}_{t_\mathrm{j}}) - \nabla\cdot(\gamma\nabla\mathrm{Q}_{t_\mathrm{j}}) = -\frac{2\omega}{2n+1}\left(\sum_{\mathrm{i}=1}^{2n}\mathrm{P}_{(\mathrm{i}-\mathrm{j})}\mathrm{Q}_{t_\mathrm{i}}\right)$$

where

$$\mathrm{P}_\mathrm{i} = \sum_{k=1}^{n} k\sin(k\omega \mathrm{i} t_1), \qquad \text{for} \quad \mathrm{i} = \{1,2n\}$$

Harmonic Balance for Navier–Stokes Equations

Using velocity matrix **u** instead of Q yields Harmonic Balance momentum equation:

$$\nabla \cdot (\mathbf{u}_{t_j} \mathbf{u}_{t_j}) - \nabla \cdot (\gamma \nabla \mathbf{u}_{t_j}) = -\frac{2\omega}{2n+1} \left(\sum_{i=1}^{2n} P_{(i-j)} \mathbf{u}_{t_i} \right)$$

General form of scalar transport equation becomes:

$$\nabla \cdot (\mathbf{u}_{t_j} Q_{t_j}) - \nabla \cdot (\gamma \nabla Q_{t_j}) = -\frac{2\omega}{2n+1} \left(\sum_{i=1}^{2n} P_{(i-j)} Q_{t_i} \right)$$

Continuity equation remains the same, with **u** replaced with its discrete counterpart $\mathbf{u}_{t_j}$:

$$\nabla \bullet \mathbf{u}_{t_j} = 0.$$

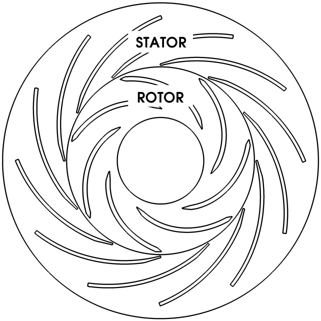
Validation of Harmonic Balance for Navier–Stokes equations

Two test cases:

- ERCOFTAC Centrifugal Pump, 2D

- OtaBm2 Pump, 3D

ERCOFTAC Centrifugal Pump



- radial inlet velocity: 11.4 m/s
- rotational speed: 2000 rpm
- k-ε turbulence model

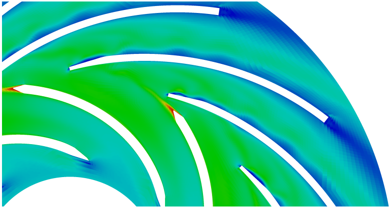
- multiple-frequency approach

$$f_{\mathrm{rotor}} = rpm/60$$
$$f_{\mathrm{stator}} = f_{\mathrm{rotor}} \cdot n_{\mathrm{rotor\ blades}}$$

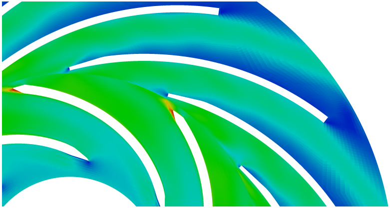
Pump global parameters comparison:

		Transient	HB, 1h	error, %	HB, 2h	error, %	MRF	error, %
$t = \frac{T}{3}$	Efficiency	89.72	88.80	1.0	89.76	0.0	89.65	0.07
	Head	81.48	81.80	0.4	80.45	1.3	84.12	3.14
	Torque	0.0297	0.0302	1.7	0.0294	0.9	0.0308	3.57
$t = \frac{2T}{3}$	Efficiency	89.92	88.78	1.3	89.81	0.1		
	Head	81.48	81.85	0.4	80.6	1.1		
	Torque	0.0296	0.0302	2.0	0.0295	0.4		
$t = T$	Efficiency	89.83	88.85	1.1	89.71	0.1		
	Head	81.49	81.79	0.4	80.39	1.3		
	Torque	0.0297	0.0302	1.6	0.0294	1.0		

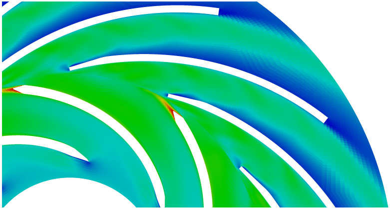
Local transient effects comparison, velocity field:



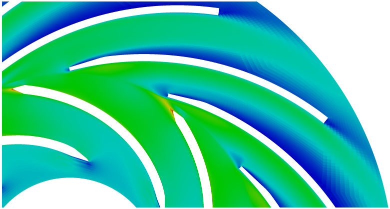
Transient simulation.



HB with 1 harmonic.

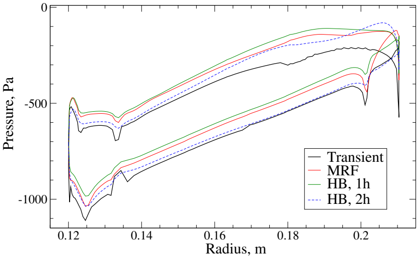


HB with 2 harmonics.

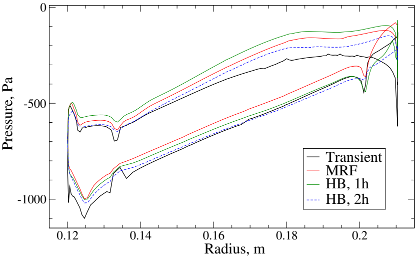


MRF simulation.

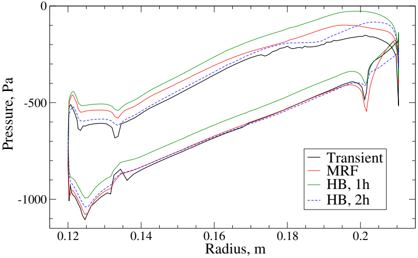
Rotor blade pressure contours at 1/3T, 2/3T and T time instants:



a) $t = T/3$.



b) $t = 2T/3$.



c) $t = T$.

CPU time comparison:

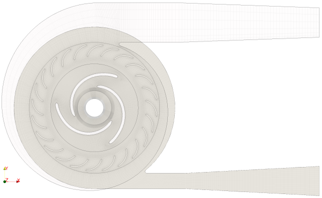
Transient	1 period $\sim$ 5 hours, 8 periods $\sim$ 40 hours
MRF	3100 iterations, $\sim$ 20 minutes
HB, 1h	3000 iterations, $\sim$ 52 minutes
HB, 2h	2400 iterations, $\sim$ 78 minutes

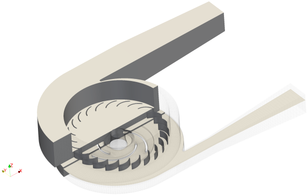
Transient simulation:

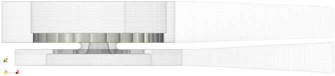
- 600 time steps per rotation
- 2 outer correctors
- 4 inner correctors

Harmonic Balance with 2 harmonics approximately **30 times faster** than transient simulation.

OtaBm2 Centrifugal Pump







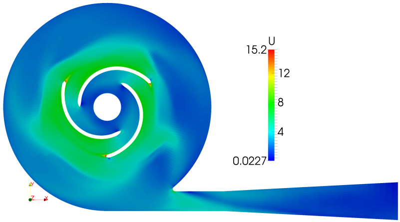
- inlet velocity: 0.933 m/s
- rotational speed: 800 rpm
- k-ε turbulence model

Pump global parameters comparison:

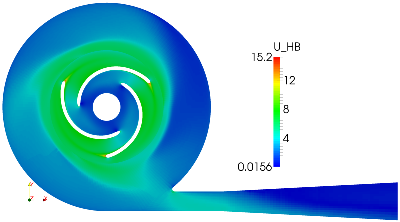
		Transient solver	HB, 1h	error, %	MRF	error, %
$t = \frac{T}{3}$	Efficiency	63.09	65.87	4.2	60.58	3.9
	Head	6.67	7.11	6.2	6.74	1.1
	Torque	19.53	19.94	2.1	20.57	5.3
$t = \frac{2T}{3}$	Efficiency	64.76	66.37	2.4	60.58	6.5
	Head	6.49	7.16	9.4	6.74	3.9
	Torque	18.54	19.93	6.9	20.57	10.9
$t = T$	Efficiency	62.02	65.99	6.0	60.58	2.3
	Head	6.75	7.12	5.2	6.74	0.1
	Torque	20.11	19.94	0.8	20.57	2.2

FSB

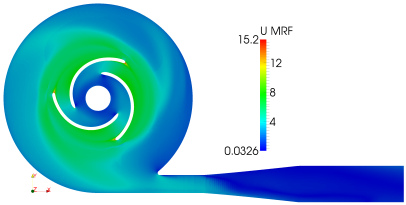
Velocity field:



Transient simulation.

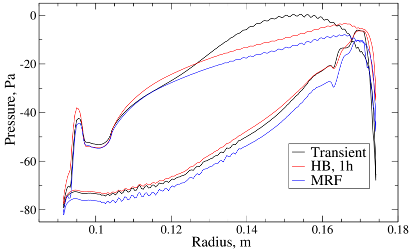


HB simulation, 1h.

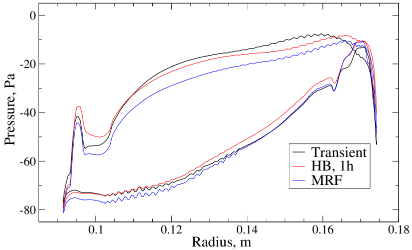


MRF simulation.

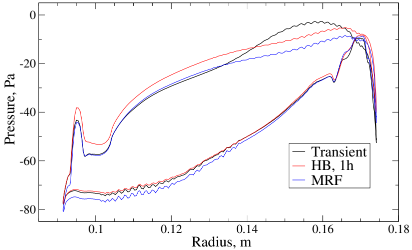
Rotor blade pressure contours at 1/3T, 2/3T and T time instants:



$t = T/3.$



$t = 2T/3.$



$t = T.$

Conclusion

- Harmonic Balance method implemented in foam-extend
- validated for scalar transport and Navier–Stokes equations, 2D and 3D
- computationally more efficient than conventional transient methods
- wide range of application

Future work:

- implicit Harmonic Balance benchmarking for turbomachinery
- full multiple-frequency approach
- multistage turbomachinery
- periodic boundary conditions

Thank you for your attention.
Questions?