

The Harmonic Balance Method for Temporally Periodic Free Surface Flows in Marine Hydrodynamics

Inno Gatin¹, Gregor Cvijetić¹, Vuko Vukčević¹, Hrvoje Jasak^{1,2}

¹Faculty of Mechanical Engineering and Naval Architecture

University of Zagreb, Croatia

²Wikki Ltd., United Kingdom

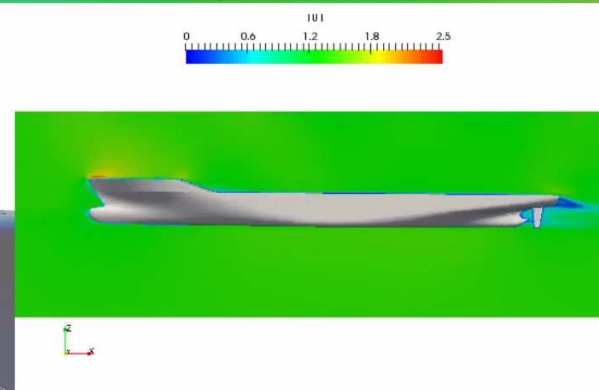
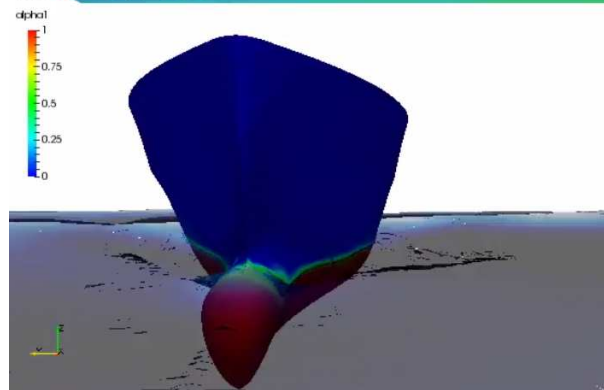
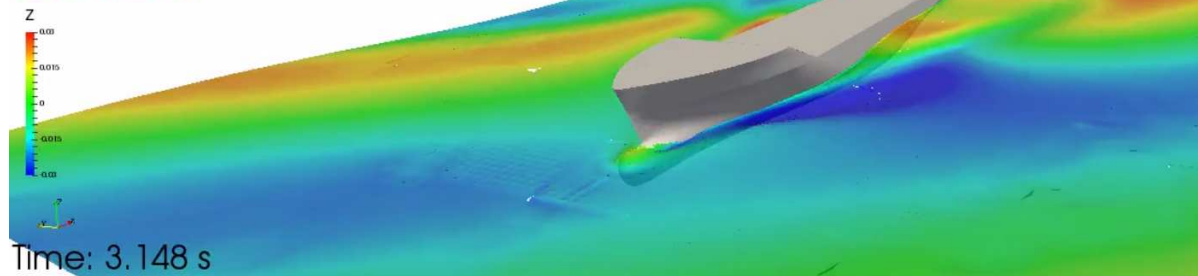
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{inno.gatin, gregor.cvijetic, vuko.vukcevic, hrvoje.jasak}@fsb.hr,

h.jasak@wikki.co.uk

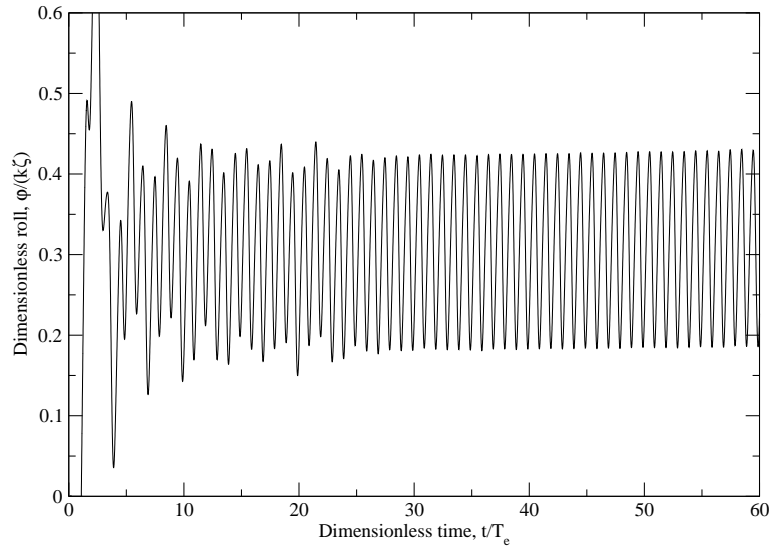
Seakeeping Transient Simulation

KCS-6DOF
swenseFoam
Bow waves

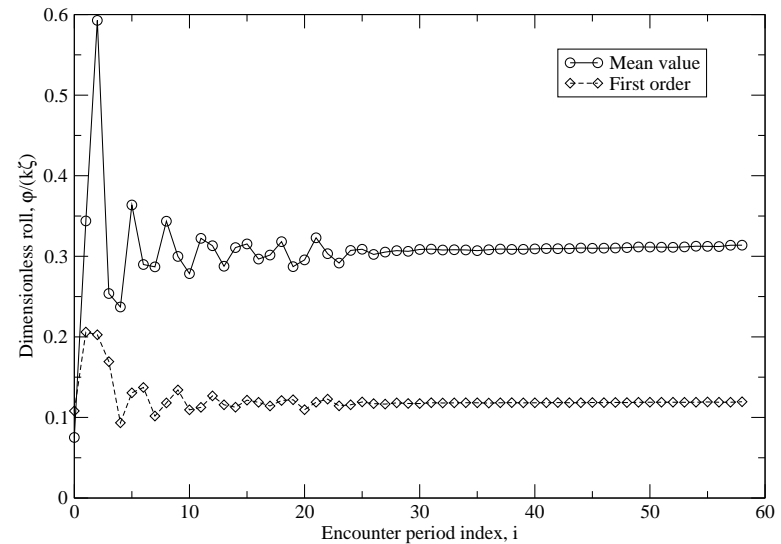


KCS 2.11-C2 seakeeping case, 45° .

Motivation



Time domain roll signal.



Moving window FFT.

- For this particular case (bow waves): **more than 20 encounter periods** needed to lower periodic uncertainty.

Idea:

- Try to solve seakeeping of the ship in the frequency domain while...
- ...preserving nonlinear, viscous, turbulent and two-phase effects.
- It turns out that evaluating convection and diffusion operators on arbitrary geometry in frequency domain is extremely difficult...

Time-spectral **Harmonic Balance** approach:

- Project a transient problem into a **set of coupled steady state problems**,
- Considerable CPU time reduction is achieved,
- Adequate for wave related phenomena such as wave-structure interaction in regular waves with well defined base frequency.

The present work utilizes existing two-phase numerical model developed in the `Naval Hydro` pack, which includes:

- Solution decomposition approach via **SWENSE** approach,
- **Ghost Fluid Method** for discretisation of interface jump conditions,
- **Implicitly redistanced Level Set** method for interface capturing,
- Domain decomposition via **implicit relaxation zones**.

Fields are expanded in Fourier series using known base frequency:

$$\phi(t) = \Phi_0 + \sum_{i=1}^N \Phi_{S_i} \sin(i\omega t) + \Phi_{C_i} \cos(i\omega t),$$

and inserted into the general transport equation:

$$\frac{\partial \phi}{\partial t} + \mathcal{R} = 0.$$

By analytically calculating temporal derivative and equating (balancing) the corresponding harmonics yields $2N + 1$ equations in the frequency domain:

$$\omega \underline{\underline{A}} \underline{\underline{\Phi}} + \underline{\underline{R}} = \underline{\underline{0}}. \quad (1)$$

The system of equations is then transformed back into the time domain by discrete Fourier Transform, yielding $2N + 1$ **coupled steady state equations**:

$$\omega \underline{\underline{E}}^{-1} \underline{\underline{A}} \underline{\underline{E}} \underline{\underline{\phi}} + \underline{\underline{R}} = \underline{\underline{0}}. \quad (2)$$

Equations are coupled via temporal source term $\omega \underline{\underline{E}}^{-1} \underline{\underline{A}} \underline{\underline{E}} \underline{\phi}$.

The coupling can be performed **explicitly**:

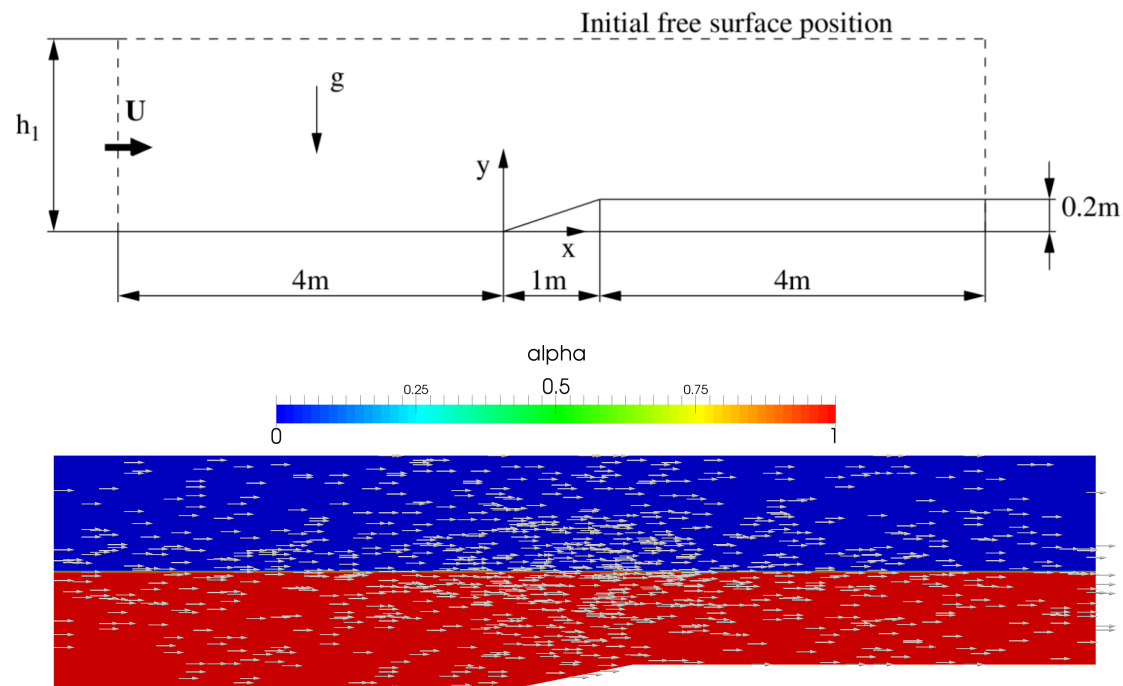
- + Up to 10 times faster than transient simulations,
- Limited to flows with dominant mean flow,
- Fine grids required to enhance convergence,
- Numerical stability is deteriorated with increasing number of resolved harmonics,
- Not adequate for wave related problems,

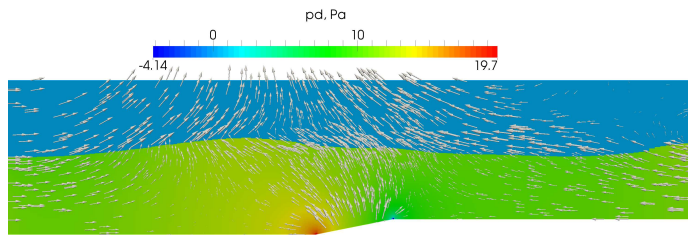
or **implicitly** using existing block matrix functionalities:

- + Stable with low (and zero) mean flows,
- + Coarser grid enhances the stability and accelerates the convergence (early design stage),
- + Increasing the number of harmonics improves convergence,
- + Adequate for wave related problems,
- Acceleration with respect to transient simulation up to ≈ 3 times.

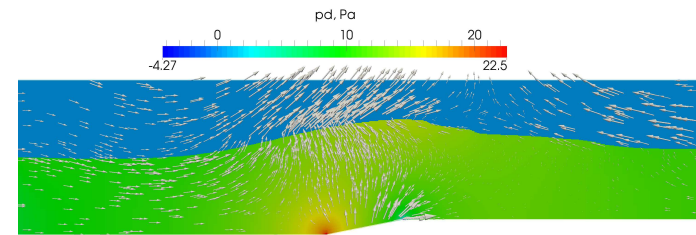
2D ramp test case

- Periodic inlet velocity boundary condition (in the x direction),
- $U_0 = 6 \text{ m/s}$, $U_1 = 1 \text{ m/s}$, $T = 0.5 \text{ s}$,
- Sensitivity studies:
 - Grid convergence study,
 - Spectral resolution convergence study,
- Result and CPU time comparison with transient simulation.

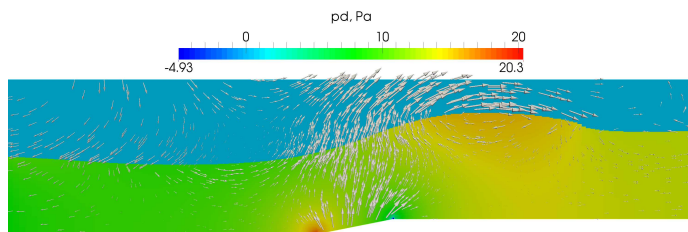




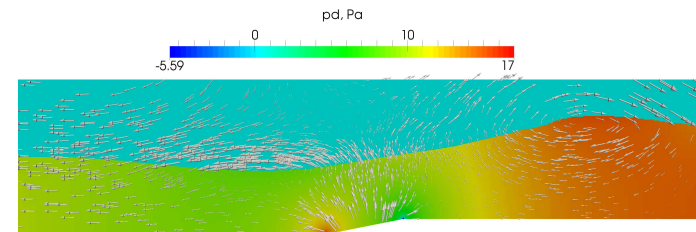
(a) $t = T/5$.



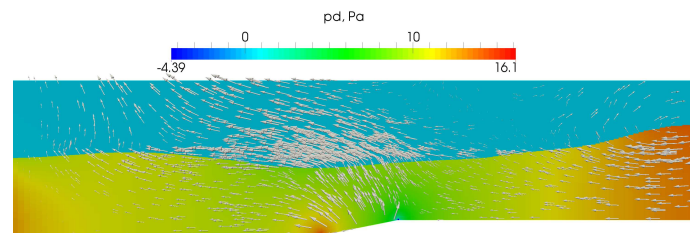
(b) $t = 2T/5$.



(c) $t = 3T/5$.



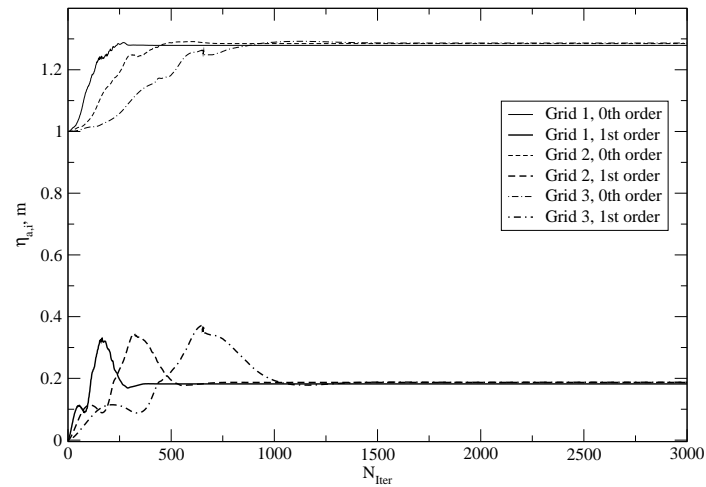
(d) $t = 4T/5$.



(e) $t = T$.

Grid convergence study

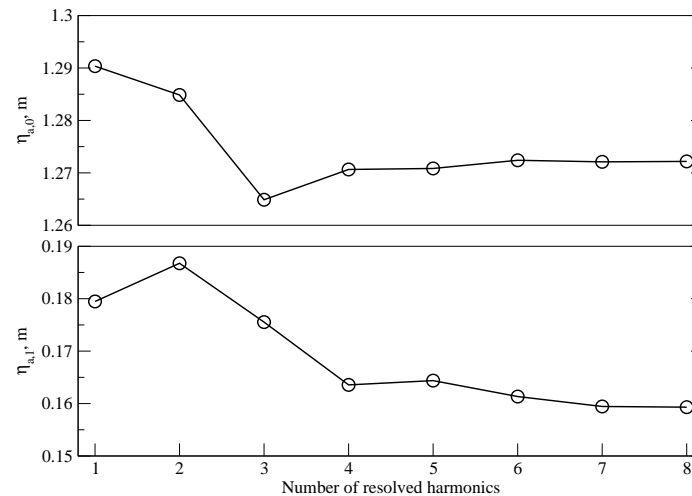
- Three grids are used with refinement ratio $r = 2$.



Order	Convergence type	p	$U_G, \%$
0	Monotone	1.750	0.085
1	Monotone	2.860	0.083
2	Monotone	2.099	1.930

Spectral resolution refinement study

- Simulations with 1, 2, 3, 4, 5, 6, 7 and 8 harmonics are performed, while 2, 4, and 8 are used for the convergence study.



Order	Convergence type	p	$U_S, \%$
0	Oscillatory	/	0.837
1	Monotone	2.450	0.896
2	Monotone	1.717	5.527

Transient simulation comparison



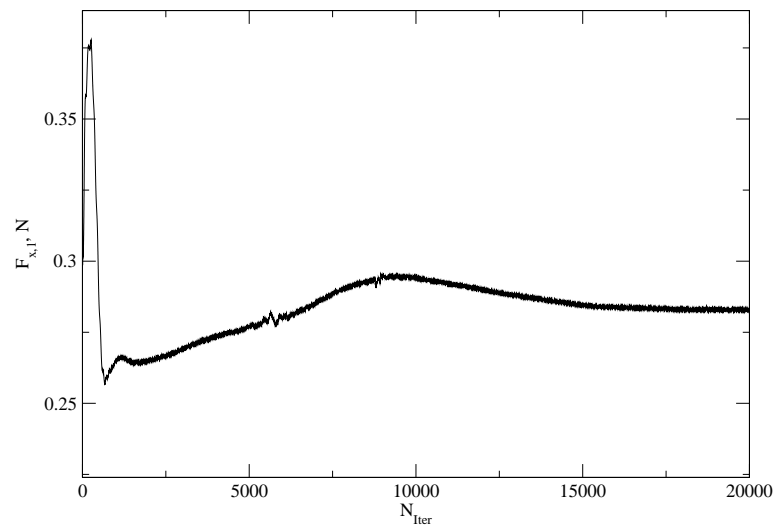
Simulation type	CPU time, s	Speed-up
Transient (10 periods)	5067	1
Explicit HB (2 harmonics)	488	10.4
Implicit HB (2 harmonics)	1851	2.7

No. Harmonics	$\eta_{a,0}$, m	$\eta_{a,1}$, m	ϵ_0 , %	ϵ_1 , %
1	1.29035	0.179472	-1.6	-15.0
2	1.28485	0.186762	-1.2	-19.6
3	1.26487	0.175538	0.4	-12.5
4	1.27065	0.163556	-0.1	-4.8
5	1.27084	0.164377	-0.1	-5.3
6	1.27240	0.161346	-0.2	-3.4
7	1.27210	0.159447	-0.2	-2.1
8	1.27218	0.159308	-0.2	-2.1

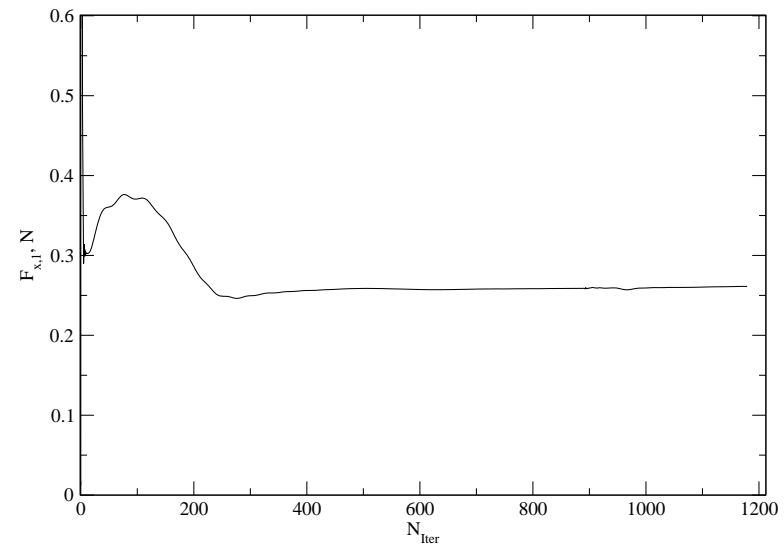
Cylinder diffraction

- Zero mean flow,
- Wave parameters: $\lambda = 0.77$ m, $ka = 0.1$,
- Cylinder diameter: $D = 0.06$ m.

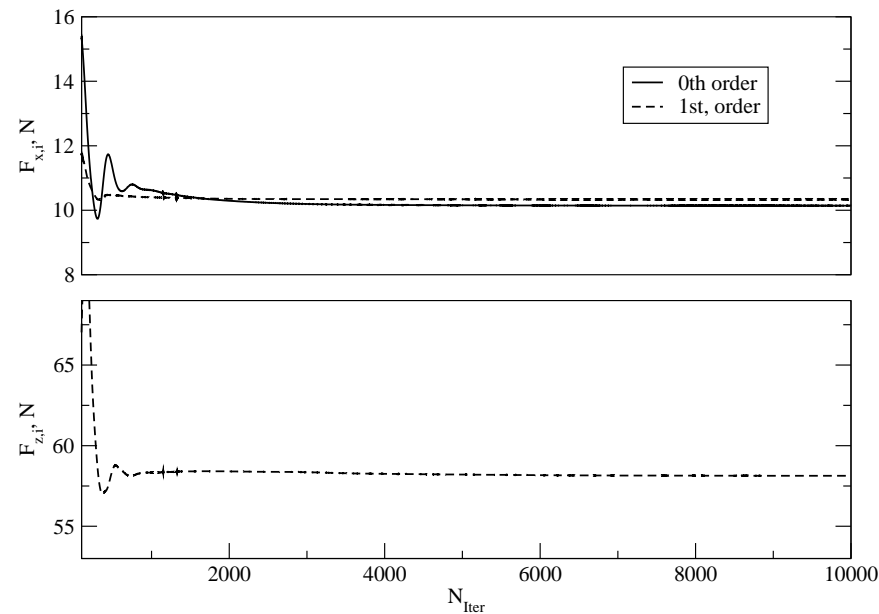
5 Harmonics



7 Harmonics



- Ship model parameters: $L = 3.05$ m,
- Froude number: $Fr = 0.28$, $U = 1.52$ m/s,
- Wave parameters: $H = 0.036$ m, $T = 1.09$ s, $\lambda = 4.57$ m,
- 2 harmonics are used in the HB simulation,
- 200 time steps per encounter wave period are used in the transient simulation.



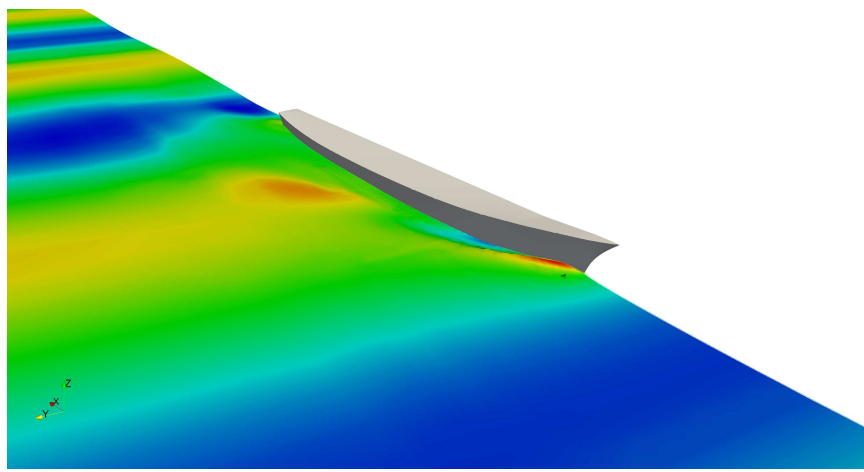
Convergence of longitudinal and vertical force harmonics.

Transient simulation comparison

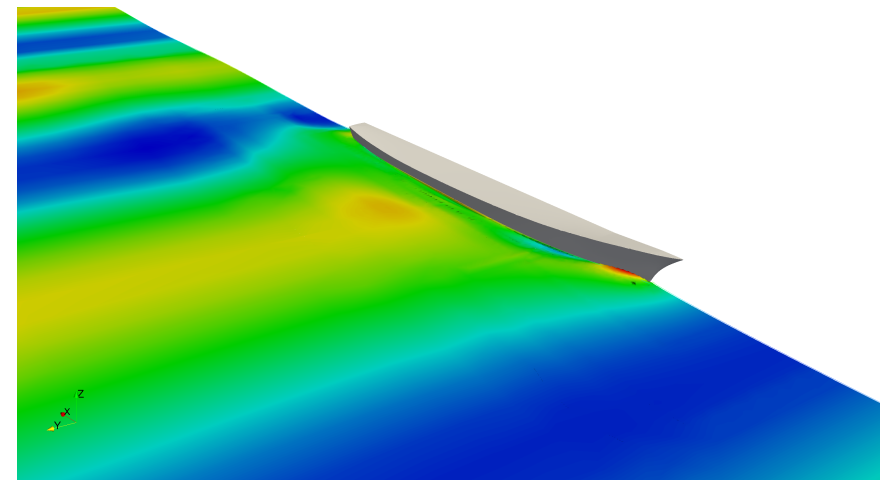


Simulation type	CPU time, h	Speed-up
Transient (20 periods)	18.86	1
Implicit HB (2 harmonics)	8.6	2.2

Item	Transient	Harmonic Balance	ϵ , %
$F_{x,0}$, N	9.20	10.14	-10.2
$F_{x,1}$, N	10.70	10.34	3.36
$F_{z,0}$, N	784.88	785.72	-0.11
$F_{z,1}$, N	62.63	58.14	7.17



Transient simulation.



HB simulation.

Sensitivity studies and comparisons with the transient solver lead to following conclusions:

- Two-phase HB method is applicable for marine hydrodynamics,
- Considerable saving in CPU time can be achieved,
- Accurate results can be achieved with less computational resources,
- Adequate for early design stages,
- Rate of convergence can be accelerated by simply increasing the number of harmonics (in the implicit formulation),
- Stable for zero mean flows!

TO-DO

- Accelerate implicit coupling by optimising the block coupled numerical framework,
- Implement a 6-DOF domain motion model in frequency domain for general seakeeping simulations,
- Efficient self-propulsion simulations using the HB method for propeller modelling.