# Acoustic analysis of a turbulent flow around a bluff body

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# Outline

- Motivations
- Overview on Aeroacoustic Analogies
- Numerical Approach
- Case of Study
- Future steps

# Motivations

Aeroacoustic study of fluid-mechanically-generated sound has become an important research endeavor because of the growing need to control transport noise.

As ship design advances, particularly with regard to structural optimisation and high speeds to meet market demands, there is a tendency for noise and vibration problems to become more pronounced.

And it follows an increasing attention on acoustic pollution and its impact on marine life.

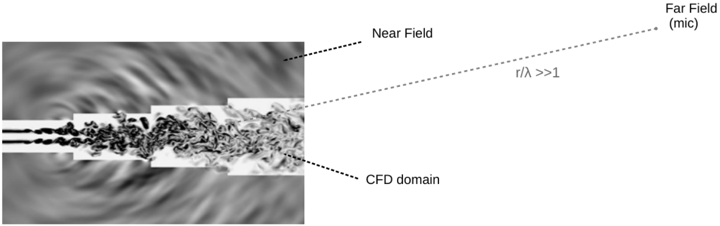
Ship underwater noise is quite unexplored research field:

when looking at the literature, it is easy to recognize a lack of both theoretical and computational models. At present, the criteria adopted to satisfy the noise emission requirements are based on empirical basis and the use of some approximated numerical procedure.

The hydroacoustic behaviour of a marine propeller is generally not investigated at design stage.

# Aeroacoustics

*Direct vs Hybrid methods*




*Free jet noise, N. Sandham, Z. Hu and C. Morfey*

## Acoustic Scales

Speed of sound c : air   340 m/s; water   1480 m/s

Human reception of frequencies f : 20 Hz - 20 kHz

Wavelength $\lambda = c/f$ (es. on air 17mm-17m)

# Aeroacoustics

## Lighthill 1952

The Lighthill equation represent a rearrangement of the fundamental conservation laws of mass and momentum into an inhomogeneous wave equation:

$$\begin{cases} \dfrac{\partial p}{\partial t} + \nabla(\rho u_i) = 0 \\ \dfrac{\partial}{\partial t}(\rho u_i) + \nabla(\rho u_i u_j + p\delta_{ij} - \tau_{ij}) = 0 \\ c_0^2 = \left(\dfrac{\partial p}{\partial \rho}\right)_{s_0} \end{cases} \Longrightarrow \quad \Box^2 p = \underbrace{\frac{\partial^2}{\partial x_i \partial x_j}(\rho u_i u_j - \tau_{ij} + (p - c_0^2\rho)\delta_{ij})}_{\text{Acoustic source}}$$

The RHS term includes all possible noise source mechanisms taking place in the flow:

- $\Box^2 = \left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)$
- the convective term, represented by Reynolds tensor $\rho u_i u_j$
- the possible deviation from isentropic behaviour $(p - c_0^2\rho)\delta_{ij}$
- viscous stresses $\tau_{ij}$

# Aeroacoustics

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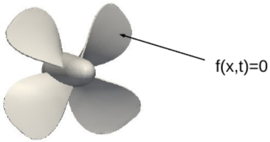
# Aeroacoustics

## Ffowcs-Williams, Hawkings 1967

The Ffowcs Williams-Hawkings equation is an extension of Lighthill work, accounting for the possible presence of a body moving in the fluid.

Such a presence is described by representing the surface $f(x,t)=0$ (on which $u_n = v_n$) as a moving discontinuity in the flow and, then, re-writing the same conservation laws in terms of generalized functions:

$$\Box^2 pH(f) = \underbrace{\frac{\partial^2}{\partial x_i \partial x_j}\{T_{ij}H(f)\}}_{\text{Vol Quadrupole source}} + \underbrace{\frac{\partial}{\partial t}\left(\rho_0 v_i \frac{\partial f}{\partial x_j}\delta(f)\right)}_{\text{Surf Monopole source}} - \underbrace{\frac{\partial}{\partial x_i}\left(P_{ij}\frac{\partial f}{\partial x_j}\delta(f)\right)}_{\text{Surf Dipole source}}$$



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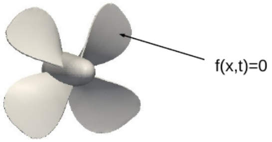


Where:

- $T_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}(\rho u_i u_j - \tau_{ij} + (p - c_0^2\rho)\delta_{ij})$
- $P_{ij} = p\delta_{ij} - \tau_{ij} - p_0\delta_{ij}$

Compared to Lighthill equation and the original flow 3D term, two additional surface 2D terms appear, known as thickness and loading noise components.

# Aeroacoustics

Previous equations are turned into an integral form by using the free-space Green function for the wave equation:

$$G(x, y; t, \tau) = \frac{\delta(t-\tau-|x-y|/c_0)}{4\pi|x-y|}$$

The solution of an inhomogeneous wave equation of the kind:

$$\Box^2 p = q(x, t)|\nabla f(x)|\delta(f)$$

is given by the following, integral form:

$$p(x, t) = \int_{\infty}^{t} \int_V G(x, y; t, \tau) q(y, \tau)|\nabla f(x)|\delta(f) dV(y) d\tau$$

The emission time $\tau = t - |x - y|/c_0$ (at which the integral has to be computed) represents the instant at which the noise is emitted from $y$ to reach the observer $x$ at time $t$.

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We get to the Formulation 1, originally developed for aeroacoustic analysis of helicopter rotors, refers to FWH 2D terms, which depend on body shape ($v_n$, thickness noise) and surface loads ($p$, loading noise)

$$4\pi p(x,t) = \frac{\partial}{\partial t}\int_S \left[\frac{\rho_0 v_n}{r|1-M_r|}\right]_\tau dS + \frac{1}{c_0}\frac{\partial}{\partial t}\int_S \left[\frac{p\, r_n}{r|1-M_r|}\right]_\tau dS + \int_S \left[\frac{p\, r_n}{r^2|1-M_r|}\right]_\tau dS$$

With $M_r = v \cdot (x-y)/c_0$.

# Aeroacoustics

The same Green function approach allows us to turn the nonlinear, Lighthill term into some 3D integrals, where the integration domain theoretically represents the whole volume of fluid V affected by body motion, but it turns out to be CPU time demanding.

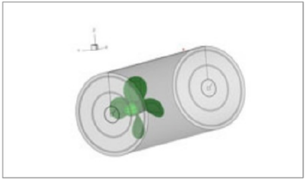
In 1997 the first numerical results coming from the FWH porous formulation were published. This formulation computes the FWH integrals on a closed domain $S_p$, outer the body souce and permeable ($u_n \neq v_n$), which embeds possible nonlinear sources.

$$4\pi p(x,t) = \frac{\partial}{\partial t}\int_S \left[\frac{\rho_0 U_n}{r|1-M_r|}\right]_\tau dS + \frac{1}{c_0}\frac{\partial}{\partial t}\int_S \left[\frac{L_r}{r|1-M_r|}\right]_\tau dS + \int_S \left[\frac{L_r}{r^2|1-M_r|}\right]_\tau dS$$

Where:

- $U_i = \left(1 - \frac{\rho}{\rho_0}\right) v_i - \frac{\rho}{\rho_0} u_i$
- $L_i = P_{ij} n_j + \rho u_i (u_n - v_n)$

By suitably fixing $S_p$, the porous equation (automatically accounting for the contribution of the nonlinear sources) can avoid any volume integration. However, an accurate solution of the fluid-dynamic problem on $S_p$ has to be available.



S.Ianniello, The Ffowcs Williams-Hawkings equation for hydroacoustic analysis of rotating bodies

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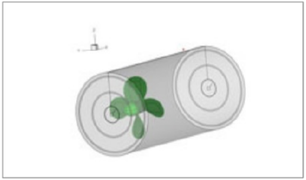
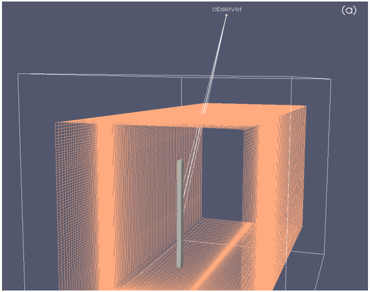
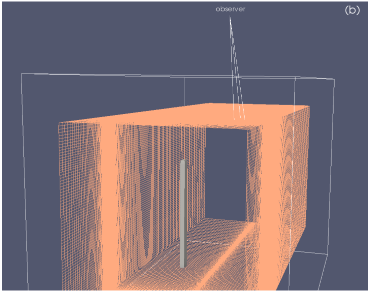


S.Ianniello, The Ffowcs Williams-Hawkings equation for hydroacoustic analysis of rotating bodies

# Numerical Approach

In order to implement formulation 1 (a) or the porous formulation (b) we need to collect the data either on the body surface or on the porous domain.




We need to know for **each cell** of the "source domain" (at **each time step**) : the flow data (p and U), the outward normal to the surface n, the distance from the observer r, the area of the cell dS. Than we integrate on the surface, and do derivative in time for some terms.

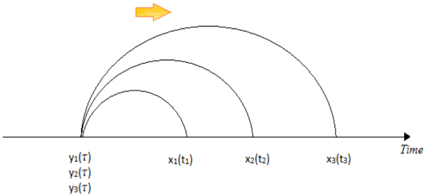
# Numerical Approach

The time $\tau$ at wich the integral has to be computed is the emission time, it represents the instant at which the noise is emitted from y to reach the observer x at time t.

$$\tau = t - \frac{|x - y|}{c_0}$$

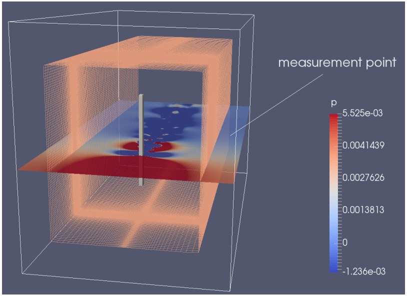
The difference between $\tau$ and t is a fundamental feature of propagation mechanisms: it is known as compressibility delay and points out that the speed of sound $c_0$ is finite.

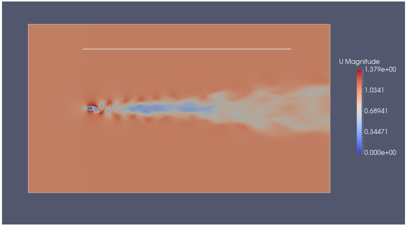
We can fix $\tau$ and move forward in time in order to determine $t$. Data-fitting is then required to rebuild the overall, resulting noise $p(x, t)$.



# Case of study I

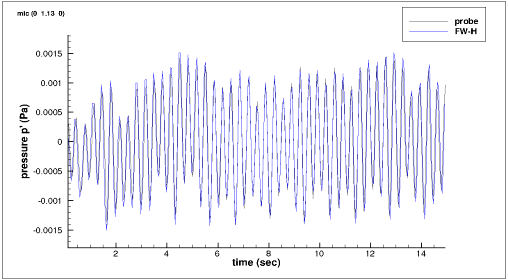
## Noise of a subsonic flow around a square cylinder

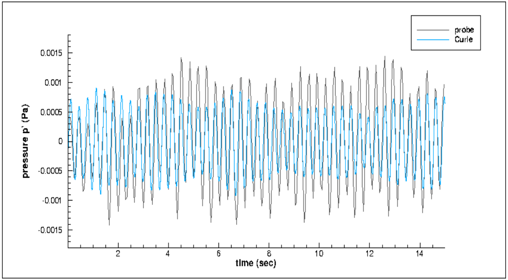


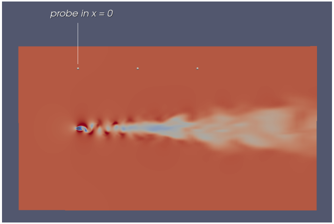




- $U_{in} = 1m/s$, $D = 0.04m$, $H = 1.2m$, $Re_D = 4000$,
- The square cylinder is situated in a tunnel $70D$ width, $80D$ hight and $125D$ long, cyclic boundary conditions are imposed on the sides of the tunnel.
- Resolved LES, PISO algorithm.

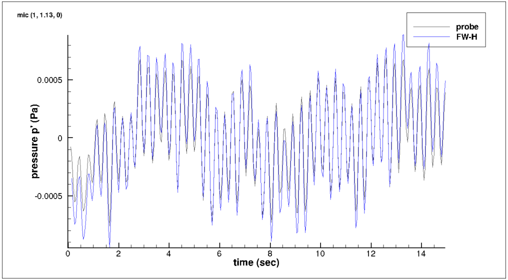
# Curle vs FW-H

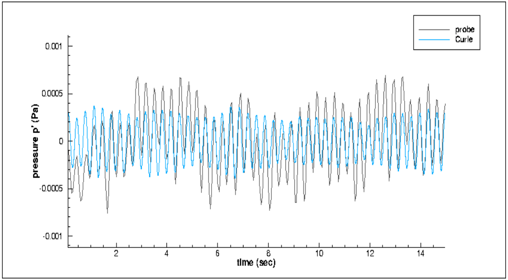

mic (0 1.13 0)
probe
FW-H
0.0015
0.001
0.0005
0
-0.0005
-0.001
-0.0015
pressure p' (Pa)
2
4
6
8
10
12
14
time (sec)

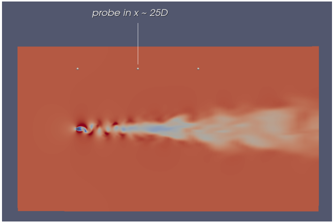

probe
Curle
0.0015
0.001
0.0005
0
-0.0005
-0.001
-0.0015
pressure p' (Pa)
2
4
6
8
10
12
14
time (sec)


probe in x = 0

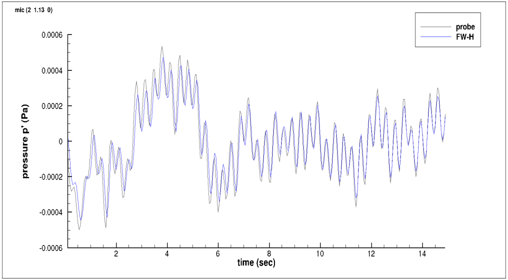
# Curle vs FW-H

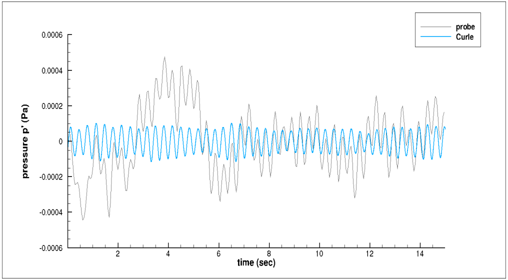

mic (1, 1.13, 0)
probe
FW-H
pressure p' (Pa)
0.0005
0
-0.0005
0
2
4
6
8
10
12
14
time (sec)

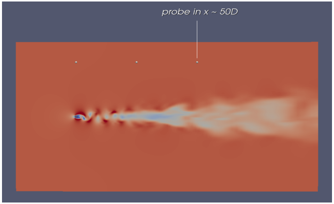

probe
Curle
pressure p' (Pa)
0.001
0.0005
0
-0.0005
-0.001
2
4
6
8
10
12
14
time (sec)


probe in x ~ 25D

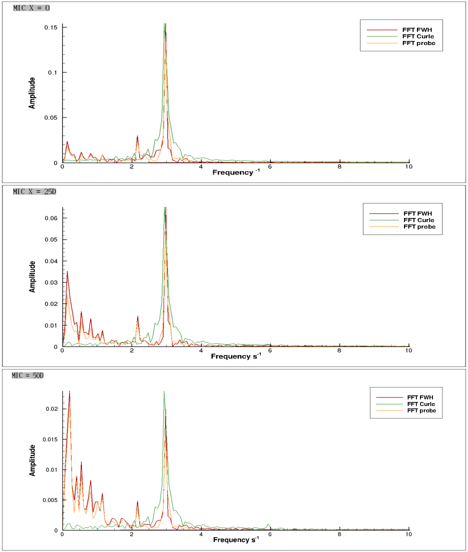
# Curle vs FW-H


mic (2 1.13 0)
probe
FW-H
pressure p' (Pa)
time (sec)


probe
Curle
pressure p' (Pa)
time (sec)


probe in x ~ 50D

# Frequency and Sound Pressure Level



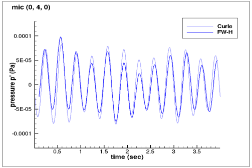

$$SPL = 20x\log(\frac{p_{rms}}{p_0})dB$$

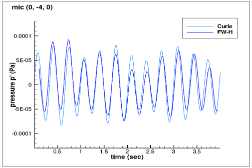
*Reference Sound Pressure

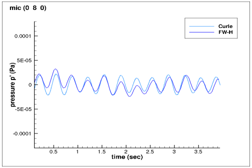
$p_0$ in air $= 2x10e^{-5}$ Pa $= 0$ dB

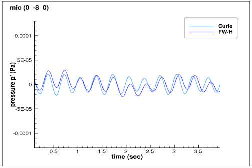
| | Curle | FW-H / probe |
|---|---|---|
| mic x=0 | 63 dB | 72 dB |
| mic x=25D | 46 dB | 57 dB |
| mic x=50D | 21 dB | 44 dB |

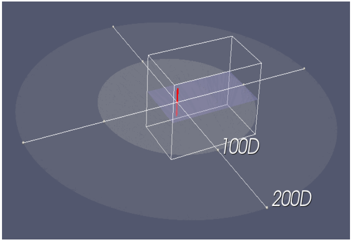
# Curle vs FW-H ⇝ Far Field



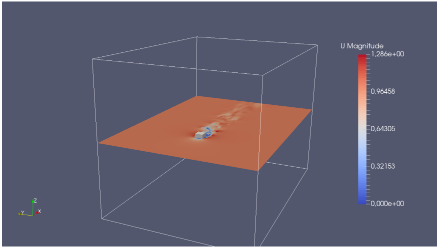



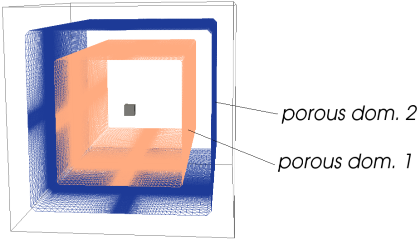



# Case of study II

Noise of a subsonic flow around a cube

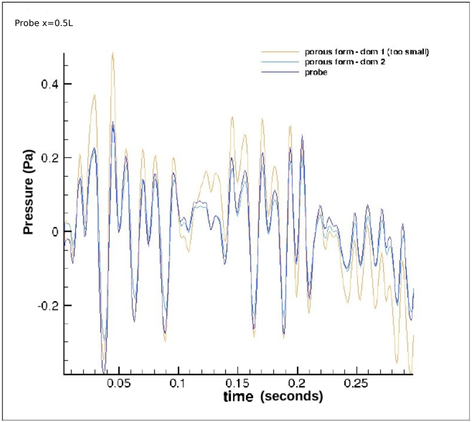


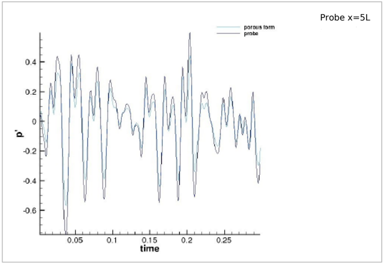



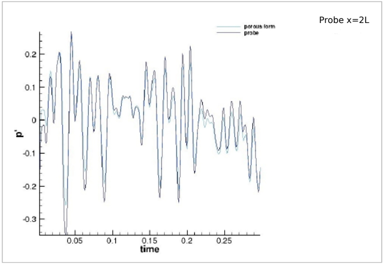

- $U_{in} = 1m/s$, $D = 0.08m$, $Re_D = 8000$,
- The cube is situated in a tunnel 20D width, 20D hight and 35D long, cyclic boundary conditions are imposed on the sides of the tunnel.
- Resolved LES, PISO algorithm.

# Case of study II

Noise of a subsonic flow around a cube



# Future steps

- Study how the 3D terms affect the far field noise

$$
\begin{aligned}
4\pi p_{NL}(x,t) = & \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\int_V \left[\frac{T_{rr}}{r|1-M_r|}\right]_\tau dV \\
& + \frac{1}{c_0}\frac{\partial}{\partial t}\int_V \left[\frac{3T_{rr}-T_{ii}}{r^2|1-M_r|}\right]_\tau dV \\
& + \int_V \left[\frac{3T_{rr}-T_{ii}}{r^3|1-M_r|}\right]_\tau dV
\end{aligned}
$$

- Try a more complex configuration, such as a system of several cubes or a flow passing through a grid
- Learn how to treat with moving boundaries, in order to study the underwater propeller noise production

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**Thank You**