IMPROVEMENTS ON THE OPENFOAM® NUMERICAL CODE FOR SIMULATION OF STEADY-STATE DIFFERENTIAL VISCOELASTIC FLOWS

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The rheological behavior of Non-Newtonian viscoelastic fluids is complex and, therefore, non-linear constitutive equations are needed to obtain realistic predictions of the fluid flow. However, the use of simple models, such as the UCM or White-Metzner models, are very challenging from the numerical point of view. Hence, this models are very suitable to test the accuracy and robustness of new developed viscoelastic numerical methods. In this work we improved the stability of the open-source differential viscoelastic numerical code in the $OpenFOAM^{®}$ framework, in order to accurately predict results in geometries with singularities or sudden changes in boundary conditions. The achievements are based on a variation of the discrete elastic-viscous stress splitting formulation (DEVSS) developed by Guénette and Fortin [1].

The basic equations to be solved are those for two or three dimensional, incompressible and isothermal laminar flow of an UCM fluid, namely the continuity equation, the momentum equation and the UCM constitutive differential equation. The continuity equation is:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

and the momentum conservation equation is:

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla \cdot (\rho\mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \boldsymbol{\tau} \tag{2}$$

together with a constitutive equation which describes the relation between the stress and the deformation rate of the fluid of interest. In the above equations, $\mathbf{u}$ is the velocity vector, ρ the density, t the time, p the pressure, and $\boldsymbol{\tau}$ the total stress tensor. The total stress tensor for the UCM fluid is only given by the elastic polymeric contribution $\boldsymbol{\tau}_P$ defined by:

$$\boldsymbol{\tau}_P + \lambda \overset{\nabla}{\boldsymbol{\tau}}_P = \eta_P[\nabla\mathbf{u} + (\nabla\mathbf{u})^T] \tag{3}$$

wherein λ is the relaxation time, η_P the polymer viscosity and $\overset{\nabla}{\boldsymbol{\tau}}_P$ denotes the upper convected time derivative defined as:

$$\overset{\nabla}{\boldsymbol{\tau}}_P = \frac{\partial \boldsymbol{\tau}_P}{\partial t} + \nabla \cdot (\mathbf{u}\boldsymbol{\tau}_P) - (\nabla\mathbf{u})^T \cdot \boldsymbol{\tau}_P - \boldsymbol{\tau}_P \cdot \nabla\mathbf{u} \tag{4}$$

The planar sudden contraction with contraction ratio H_1/H_2 of 4:1 (upstream thickness of $2H_1 = 0.04$ m and downstream thickness of $2H_2 = 0.01$ m) was chosen as first test geometry (Fig. 1). The quantitative comparison of the corner vortex

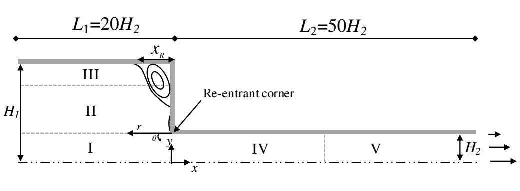

Figure 1: Schematic representation of the 4:1 planar contraction: geometry and blocks used to generate the mesh.

size is made measuring its non-dimensional length $X_R = x_R/H_2$ (see Fig. 1). The mesh refinement technique allowed to apply Richardson's extrapolation for the X_R value using the three finest meshes. Table 1 compares the results obtained with the developed code and the ones of Alves et al. (2000) [2]. The X_R results in the most refined mesh (Mesh 5), obtained with the developed code, have a difference below approximately 5% from the extrapolated value.

The second case study refers to the plane flow past a circular cylinder placed perpendicularly at the centreline of a channel (see Fig. 2). The blockage ratio in this study, defined as the ratio of cylinder radius R to channel half-height h, is $\beta = 0.5$. The results of the dimensionless drag coefficient C_D, resulting from surface integration of the stress and pressure fields around the cylinder, are listed in Table 2 and some data from Alves et al. (2001) [3] is given for comparison. Again, accurately results were obtained with a maximum error of 1% in M120 for $De = 0.6$.

Table 1: Dimensionless length of primary vortex (X_R) as a function of Deborah number and mesh for UCM fluid. Comparison between developed code and Alves et al. (2000) [2] results.

Developed code							
De	Mesh 1	Mesh 2	Mesh 3	Mesh 4	Mesh 5	Extrapolated	Difference (%)
0	1.436	1.475	1.477	1.479	1.479	1.479	0.0003
1	1.374	1.378	1.349	1.330	1.321	1.314	0.6
2	1.301	1.285	1.176	1.113	1.083	1.056	2.5
3	1.305	1.290	1.054	0.928	0.881	0.854	3.2
4	1.402	1.396	1.014	0.803	0.735	0.702	4.7
5	1.530	1.524	1.037	0.709	0.622	0.591	5.3
Alves et al. (2000)							
De	Mesh 1	Mesh 2	Mesh 3	Mesh 4	Extrapolated	Difference (%)	
0	1.472	1.488	1.494	1.495	1.496	0.1	
1	1.349	1.371	1.349	1.339	1.335	0.3	
2	1.631	1.259	1.154	1.118	1.105	1.2	
3	1.517	1.266	1.014	0.946	0.923	2.5	
4	1.644	1.337	0.987	–	0.87	13.4	
5	1.687	1.517	1.127	–	0.997	13	

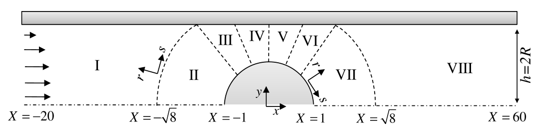

Figure 2: Schematic representation of the flow past a circular cylinder: geometry and blocks used to generate the mesh.

Table 2: Drag coefficient (C_D) as a function of Deborah number and mesh for UCM fluid. Comparison between developed code and Alves et al. (2001) [3] results.

	Developed code			**Alves et al. (2001)**			**Errors (%)**		
De	M30	M60	M120	M30	M60	M120	M30	M60	M120
0	132.071	132.435	132.503	132.23	132.342	132.369	0.1	0.1	0.1
0.3	107.956	108.435	108.891	–	108.515	108.614	–	0.1	0.3
0.6	93.142	92.828	93.197	–	92.277	92.298	–	0.6	1.0
0.9	87.182	87.244	87.242	–	87.395	87.218	–	0.2	0.03

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