THE HARMONIC BALANCE METHOD FOR TEMPORALLY PERIODIC FREE SURFACE FLOWS IN MARINE HYDRODYNAMICS

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The Harmonic Balance Method (HBM) for temporally periodic, non–linear, turbulent, free surface flows is presented in this work. The methodology is primarily targeted to efficient simulations related to wave–structure interaction in naval and offshore hydrodynamics.

Transient flows in marine hydrodynamics are often periodic, e.g. due to ocean waves (wave propagation and diffraction, seakeeping of a ship) and rotating propellers. Such flows often have well–defined base frequency: wave frequency or rotational frequency of the propeller. In fully non–linear, two–phase state–of–the–art CFD algorithms, such flows are almost exclusively resolved in time domain. Transient simulations usually require a large number of periods in order to achieve harmonically steady (purely oscillatory) solution. Due to its spectral decomposition, HBM allows us to efficiently model flow effects up to a specified order, without performing a fully transient simulation. Hence, a substantial performance improvement is expected, with almost negligible decrease in accuracy for flows with a well–defined base frequency. Due to the steady–state mathematical formulation of the HBM, authors believe that the method is highly suitable for adjoint optimisation regarding seakeeping of marine objects in the ship–building industry. This suitability has been recently confirmed by Huang and Ekici [1], who developed an adjoint shape optimisation tool based on the HBM for turbomachinery applications.

The HBM [2] has been originally developed to tackle periodic single–phase turbomachinery flows in an efficient way. This paper presents an extension of the single-phase HBM [3, 4] to two–phase free surface flows, comparing the results and computational efficiency with a transient solution. The implementation is carried out in a second–order accurate, polyhedral Finite Volume framework developed within foam-extend, a community driven fork of the OpenFOAM software.

In the HBM, a transient governing equation set is replaced with a specified number of coupled steady state problems, each represented by an equation for a unique time instant. The method simulates a periodic flow by evaluating the temporal derivative via spectral decomposition, yielding flow solution at discrete instants in time simultaneously. Multi–mode transformation from a transient to a set of coupled steady state problems is achieved by a Fourier transform, assuming temporally periodic flow. The accuracy of the model is controlled by a specified number of harmonics to allow efficient capturing of higher order flow effects. Generally, specified number of harmonics n, yields solutions at $2n + 1$ discrete time instants (n for real and imaginary parts and 1 for the mean component). As an example, HBM simulation with $n = 1$ yields solution at $t = 1/3T, 2/3T$ and T, where T is the specified period. As a post processing step, the three solutions for $n = 1$ can be used to transform the results into frequency domain, up to first order.

In the present study, HBM is applied to Navier-Stokes equations and Level Set interface capturing equation, yielding a coupled set of two-phase flow equations for discrete instants of time within one period. In addition, SWENSE decomposition is used to facilitate incident wave propagation.

Preliminary results are presented for a 2D free–surface flow over a ramp, Figure 1, where the inlet free surface height, h_1 is set to 1 m. $\vec{U}$ is the periodic inlet velocity with mean value $\vec{U}_0 = (6, 0, 0)$, first order amplitude $\vec{U}_1 = (1, 0, 0)$ and frequency $f = 0.5$ Hz. Density ratio of water and air is 1000, and the CFD domain extends 1 m above the initial free surface position. Gravity is set to $\vec{g} = (0, -9.81, 0)$, where the periodic inlet velocity causes non–linear wave propagation, where non–linear higher order effects are more pronounced after the ramp.

HBM simulations are performed with 1 and 2 harmonics, where the periodically steady state solution is achieved within 800 iterations. In contrast, transient simulation is run for 10 periods, totalling 8000 time steps with time step size of 0.0025 s. Wave gauges are positioned at 2, 4, 6 and 8 m from the inlet, and the comparison of results is briefly shown in Figure 2, where the HBM solution (obtained as a steady state solution for one period) is extended to all periods. HBM results show good agreement with transient simulation results, where the HBM simulation with 2 harmonics captures second order effects, yielding better overall agreement with the transient solution. CPU time for the transient simulation (10 periods) is 5067 s, while the HBM CPU times are 271 and 488 s, for 1 and 2 harmonics, respectively. Considering that the transient simulations regarding more complex flows (seakeeping of a ship) require more than 10 periods in order to get correct periodically averaged results, a considerable speed up factor may be expected.

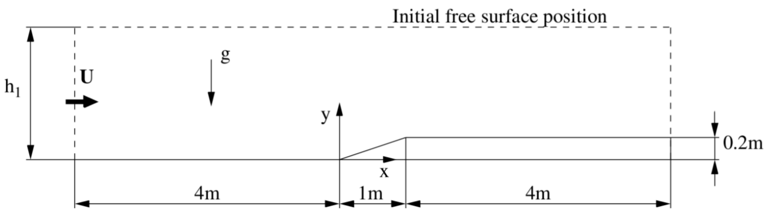
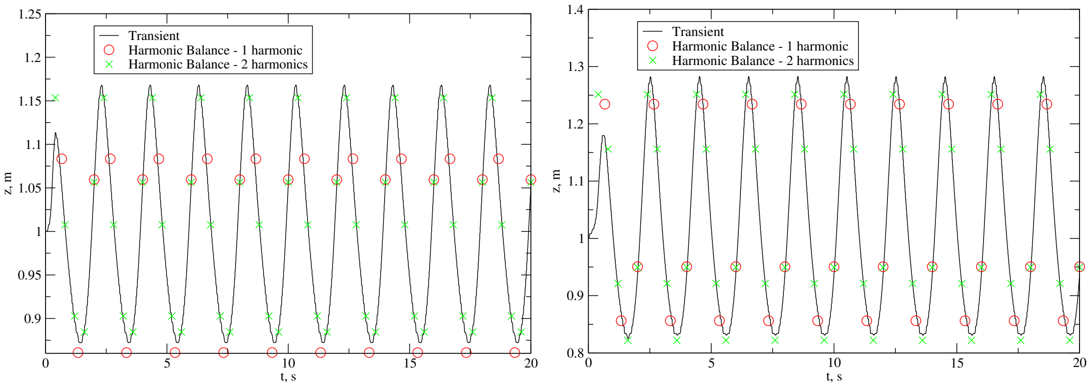
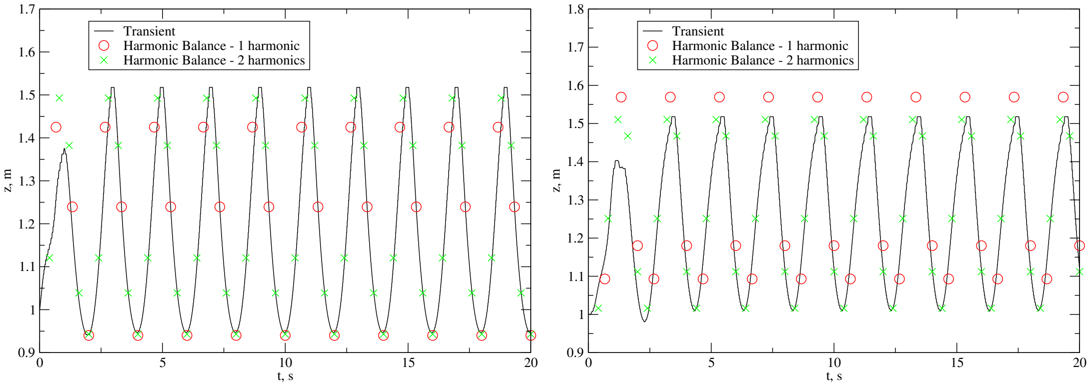


Figure 1: Set–up of a 2D ramp test case.



(a) Wave elevation 2 m downstream from inlet, **(b) Wave elevation 4 m downstream from inlet,**



(c) Wave elevation 6 m downstream from inlet, **(d) Wave elevation 8 m downstream from inlet,**

Figure 2: Free surface elevation comparison between HBM and transient simulations.

Additional validation and verification cases regarding wave propagation and diffraction are going to be performed, assessing the trade–off between accuracy and CPU efficiency of the presented approach compared to transient simulations.

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