A FULLY IMPLICIT DISCRETIZATION OF THE DIFFUSION IN OPENFOAM

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One of the main advantages of the Finite Element Method (FEM) and the Finite Volume Finite Element Method (FVFEM) over the cell centered Finite Volume Method (FVM) is that their discretization of the diffusion term generally yields a complete linearization i.e. is fully implicit while for the FVM the discretization of the diffusion operator for non-orthogonal meshes or for anisotropic diffusion will generally lead to a partial linearization with an implicit part and an explicit part also known as the non-orthogonal term. The presence of this explicit part is generally detrimental to the robustness and accuracy of the method especially when dealing with highly non-orthogonal and skewed elements or highly anisotropic diffusion coefficients.

In this paper we propose a novel approach for the discretization of the diffusion term that allows for a robust, accurate and fully implicit treatment resulting in good convergence for all types of meshes.

Introduction

The Finite Volume Method (FVM) is one of the most popular numerical techniques in Computational Fluid Dynamics (CFD) [1,2,3]. In fact the leading commercial CFD codes [4,5,6] and the majority of open source CFD codes [7,8] use the FVM for the discretization of their equations. This popularity is due to a number of properties, which include conservativeness (i.e., conservation of quantities is enforced for coarse and fine meshes alike), the ability to define biased discretization profiles (very useful for the treatment of convection), and the ability to work with two- and three-dimensional elements of any polygonal shape, without the need for shape functions.

Despite these desirable attributes, the FVM suffers a serious drawback when compared to the Finite Element Method (FEM). This shortcoming is related to the standard treatment of the diffusion term whereby diffusion term is split into an implicit (aligned, orthogonal) and an explicit (non-orthogonal, cross-diffusion) component, with the value of the latter added to the right hand side of the discretized equation. This leads to a breaks down in terms of implicitness on non-orthogonal meshes and/or for anisotropic diffusion coefficients, under certain circumstances there is even a loss of monotonicity and drastic reduction in stability [9], as can be demonstrated in a variety of cases involving anisotropic diffusion coefficients, or grids that are highly skewed and non-orthogonal [10,11,12,13,14]

LePotier [15] and later Lipnikov et al. [16,17] have tried to address these problems for the case of high anisotropic diffusion problem, however their methods still presented a variety of short-comings. In this paper the two-flux nonlinear diffusion method of Lipnikov [18] is reformulated and modified to yield a fully implicit diffusion scheme that for a simple and implicit treatment of the diffusion term for non-orthogonal grids and anisotropic coefficients.

The method denoted by the "Modified Implicit Nonlinear Diffusion" (MILD) scheme yields diffusion flux of the form

$$\overline{\nabla\phi_f}\cdot\mathbf{S}'_f=\left(\frac{E'_f}{d_{CF}}+\frac{\phi_{N_C}T'_f}{\phi_{N_C}d_{FN_F}+\phi_{N_F}d_{CN_C}}\right)\phi_F-\left(\frac{E'_f}{d_{CF}}+\frac{\phi_{N_F}T'_f}{\phi_{N_C}d_{FN_F}+\phi_{N_F}d_{CN_C}}\right)\phi_C \qquad (1)$$

in which the non-orthogonal or cross diffusion contribution is inherently accounted for.

In equation (1), $\mathbf{S}'_f$ arise from a simple reformulation of the flux to allow a simple treatment for anisotropic coefficients, namely

$$\begin{aligned}-\mathbf{K}_f\overline{\nabla\phi_f}\cdot\mathbf{S}_f&=-\overline{\nabla\phi_f}\cdot\left[\mathbf{K}_f\right]^T\mathbf{S}_f\\&=-\overline{\nabla\phi_f}\cdot\mathbf{S}'_f\end{aligned} \qquad (2)$$

MILD was implemented within OpenFOAM and its performance evaluated for a number of test cases. The scheme was shown to be 2nd order accurate as can be seen in Figure 1 that shows the error as a function of grid size for both quadrilateral and triangular elements for a simple diffusion problem with exact solution.

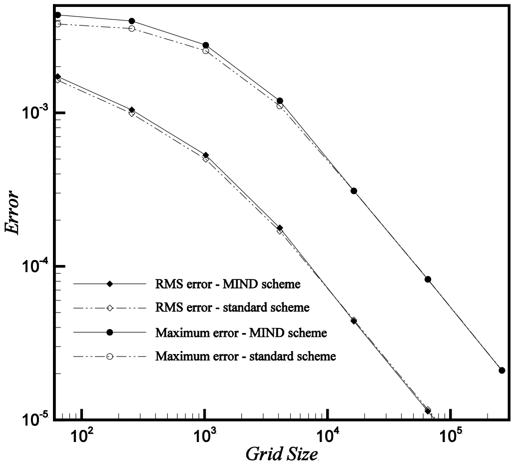

Figure 1: Title of figure

MILD was also shown to behave monotonously even for highly anisotropic coefficients on bad quality meshes as shown in Figure 1 for the Keyhani an Polehn [19] test problem.

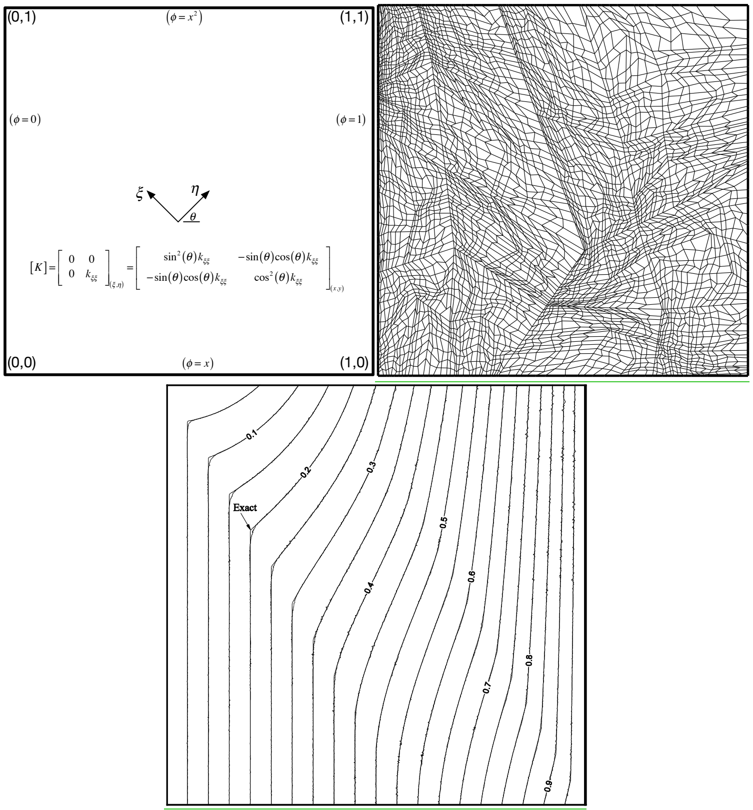

Figure 2: test case 1 with 64x64 elements and solution

A detailed derivation of MILD and a performance analysis of results on additional test cases including convergence behaviour will be presented at the workshop

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References

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[1] S. Patankar, Numerical Heat Transfer and Fluid Flow, Hemisphere Publishing, 1980

[2] Ferziger J., Peric M., Computational Methods for Fluid Dynamics 3rd Edition by, Springer , 2002

[3] F. Moukalled, L. Mangani, M. Darwish, The Finite Volume Method in Computational Fluid Dynamics: An advanced Introduction with OpenFOAM® and Matlab®,, Springer, 2015.

[4] ANSYS Fluent, http://www.ansys.com

[5] Star-CD, http://www.cd-adapco.com/products/star-cd®

[6] Numeca, http://www.numeca.com/en

[7] OpenFOAM, www.openfoam.org

[8] SU2, su2.stanford.edu

[9] J.M. Nordbotten, I. Aavatsmark, G.T. Eigestad, Monotonicity of control volume methods, Numer. Math. 106 (2) 255–288, 2007.

[10] M. Darwish, and F. Moukalled, A Compact Procedure for Discretization of the Anisotropic Diffusion Operator. Numerical Heat Transfer, Part B: Fundamentals, 55(5), 339–360, 2009

[11] S. Wang, Solving Convection-Dominated Anisotropic Diffusion Equations by an Exponentially Fitted Finite Volume Method:, Computers and Mathematics, with Applications, vol. 44, pp. 1249-1265, 2002

[12] P.A. Jayantha, I.W. Turner, On the use of surface interpolation techniques in generalised finite volume strategies for simulating transport in highly anisotropic porous media, J. Comput. Appl. Math. 152 (1–2) 199–216, 2003.

[13] E. Bertolazzi, G. Manzini, A second-order maximum principle preserving finite volume method for steady convection–diffusion problems, SIAM, J. Numer. Anal., vol. 43 no. 5 pp. 2172–2199, 2006

[14] E. Eymard, T. Gallouêt, R. Herbin, A finite volume scheme for anisotropic diffusion problem, Comptes Rendus Acad. Sci., Mathématiques, vol 339, no. 4, pp 299–302, 2004

[15] C. LePotier, Schema volumes finis monotone pour des operateurs de diffusion fortement anisotropes sur des maillages de triangle non structures, C. R. Math. Acad. Sci. Paris 341 787–792, 2005

[16] K. Lipnikov, D. Svyatskiy, M. Shashkov, Y. Vassilevski, Monotone finite volume schemes for diffusion equations on unstructured triangular and shape-regular polygonal meshes, J. Comp. Phys. 227, 492–512, 2007.

[17] K. Lipnikov, D. Svyatskiy, Y. Vassilevski, Interpolation-free monotone finite volume method for diffusion equations on polygonal meshes,Journal of Computational Physics 228, 703–716, 2009

[18] K. Lipnikov, D. Svyatskiy, Y. Vassilevski, Minimal stencil finite volume scheme with the discrete maximum principle, Russ. J. Numer. Anal. Math. Modelling, vol. 27, No. 4, pp. 369–385, 2012

[19] M. Keyhani, P.A. Polehn, Finite Difference Modeling of Anisotropic Flows, ASME J. Heat Trans., vol. 117, pp. 458-464, 1995