SIMULATION OF VISCOELASTIC SINGLE- AND TWO-PHASE FLOWS AT HIGH WEISSENBERG NUMBER USING A GENERIC NUMERICAL STABILIZATION FRAMEWORK

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The numerical simulation of viscoelastic flows in complex geometries is challenging due to the presence of geometric singularities and sharp stress boundary layers which occur in the high Weissenberg number limit [1]. The 'High Weissenberg Number Problem' (HWNP) has been a major issue in computational rheology for more than four decades [2]. It refers to the incapability of numerical methods to resolve these geometric singularities and boundary layers, causing all computations to break down at relatively low critical values of the fluid elasticity. Although a complete solution is not known until today, several effective stabilization methods have been developed to cope with the HWNP [3, 4, 5]. However, there is a multitude of constitutive models describing viscoelastic material behavior, with which the stabilization approaches are to be combined. This combination has been realized in a general way by deriving model-independent forms of the stabilized equations.

We have developed a robust finite volume method for viscoelastic single- and two-phase flow analysis on general unstructured meshes. It is build upon a new general-purpose stabilization framework for high Weissenberg number flows. The numerical framework provides full combinatorial flexibility between different kinds of rheological models on the one hand, and effective stabilization methods on the other hand. A special face interpolation technique is employed to the cell-face interpolation of the stress in the divergence operator of the momentum balance to remove the decoupling between the velocity and stress fields, arising from the collocated variable arrangement. The discretized system of equations is solved in a segregated solution approach. The numerical methods have been implemented by massive use of generic C++ template programming, runtime polymorphism and overloading.

By means of evaluations of established benchmark-tests, such as the entry-flow of Oldroyd-B and PTT fluids through a 4:1 contraction, we demonstrate that the numerical methods are robust over a wide range of Weissenberg numbers and significantly alleviate the HWNP. The accuracy of the results is evaluated in a detailed mesh convergence study. Having a set of different stabilization approaches available on the same computational platform, we quantitatively investigate their impact on mesh-convergence and mesh-sensitivity, thus benchmarking the numerical code and yielding new understanding of viscoelastic entry-flows at higher Weissenberg numbers.

Direct numerical simulations (DNS) are performed to study the dynamics of a rising gas bubble in a viscoelastic liquid. The polymeric solution is modeled by the PTT constitutive equations. The mathematical formulation of the two-phase flow problem is derived by conditionally averaging the local instantaneous transport equations and applying a volume of fluid (VoF) model to obtain a closed-form one-field representation. The numerical stabilization framework is used for the averaged constitutive equations to assemble and solve numerically more robust representations for high Weissenberg number (HWN) flows. We investigate characteristic flow phenomena that occur at certain critical bubble volumes, i.e. the cusp-shaped tailing edge, the negative wake, and the jump discontinuity in the steady-state rise velocity.

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