SOLVING NUMERICAL STABILITY PROBLEMS IN MULTIPHASE FLOWS WITH LARGE INTERPHASE HEAT TRANSFER

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Heat transfer in multiphase flows is important in many applications as, for example, processes with phase change, reactive flows and hydrate formation. In systems containing small particles, the heat transfer between the disperse and continuous phases can be very fast and the corresponding energy exchange term in the governing equations can cause numerical instabilities when solving the energy equations for these phases. For low-speed flows of incompressible phases with no phase change, the energy equation for each phase in a multifluid model can be given in terms of the phase temperature by [1]:

$$\frac{\partial(\rho_\alpha\, Cp_\alpha\, r_\alpha\, T_\alpha)}{\partial t} + \nabla \bullet (\rho_\alpha\, Cp_\alpha r_\alpha T_\alpha\, \mathbf{u}_\alpha) = \nabla \bullet (r_\alpha \boldsymbol{q}_\alpha^{eff}) + r_\alpha \boldsymbol{\tau}_\alpha^{eff} : \nabla \mathbf{u}_\alpha + Q_{I,\alpha} \tag{1}$$

where the interphase heat transfer, $Q_{I,\alpha}$ is given by [1]:

$$Q_{I,\alpha} = \sum_{\beta=1}^{N} Q_{\alpha\beta}; \quad Q_{\alpha\beta} = h_{\alpha\beta} A_{\alpha\beta}\, (T_\alpha - T_\beta) \tag{2}$$

where h is the transfer coefficient between α and β phases and $A_{\alpha\beta}$ is the corresponding interphase heat transfer area.
The characteristic thermal response time of a particle with diameter d_α immersed in a continuous phase β, is proportional to the square of the particle diameter [2]:

$$\tau_T = \frac{\rho_\alpha\, Cp_\alpha d_\alpha^2}{12 \kappa_\beta} \tag{3}$$

where ρ, Cp and κ are, respectively, the density, specific heat capacity and thermal conductivity. For small particles, the value of the characteristic thermal response time can be very small, the time step used in the simulation should also be very small to properly capture the dynamics of the interphase heat transfer. However, multiphase flows are usually multiscale problems, and several time scales do exist. If time step limitation posed by the dynamics of the interphase heat transfer has to be enforced, the simulation can become prohibitively time consuming.
This work developed and tested a semi-implicit coupling scheme to solve an multifluid model with strong interphase heat transfer. It was shown that the proposed scheme ensure numerical stability even for time integration step much greater than the particle thermal characteristic time.
The studied case consists of an adiabatic closed rectangular cavity where heat transfer occurs between a dispersed air phase and a continuous heated water phase. Initially, air is at T_a = 200 K with a phase fraction of r_a = 0.02 and bubble diameter of d_a = 1 mm, whereas water is at T_w = 300 K with a phase fraction of r_w = 0.98. Due to the small phase fraction and heat capacity of air, its temperature rises sharply to an equilibrium temperature close to the initial water temperature (T_{eq} = 299.9995 K). The thermal characteristic time of this case is $\tau_T = 0.1622$ ms.
Figure 1(a) shows the time evolution of the air temperature for a time integration step of 10^{-5} s, which is about ten times smaller than τ_T, being able to capture the dynamics of the interphase heat transfer. As expected, T_a reached T_{eq} in about 5-6 τ_T Increasing the time step to 1 s, the solution for T_a presented strongly numerical oscillations and reached a wrong equilibrium temperature, as shown in Figure 1(b).
The proposed scheme to avoid this numerical instability consists of adding a stabilization term in the energy equation both implicitly and explicitly as shown by:

$$\frac{\partial(\rho_\alpha\, Cp_\alpha\, r_\alpha\, T_\alpha)}{\partial t} + \nabla \bullet (\rho_\alpha\, Cp_\alpha r_\alpha T_\alpha\, \mathbf{u}_\alpha) = \nabla \bullet (r_\alpha \mathbf{q}_\alpha^{eff}) + r_\alpha \boldsymbol{\tau}_\alpha^{eff} : \nabla \mathbf{u}_\alpha + Q_{I,\alpha} + (K_\alpha\, T_\alpha^* - K_\alpha\, T_\alpha) \tag{4}$$

where $K_\alpha = \sum_{\beta=1}^{N} h_{\alpha\beta} A_{\alpha\beta}$ and T_α^* is the phase temperature at the previous iteration or time. The difference between the two added terms exists only in the discretized form during the iterations [3]. This scheme increases the diagonal dominance of the discretization matrix of the Eq. 4.
Applying this scheme to the test case using the previous time value for T_a^* without any inner convergence loop, the behavior has become asymptotic and without oscillation, but that energy was not conserved as the equilibrium temperature is not correct, as can be seen in Figure 1(c).

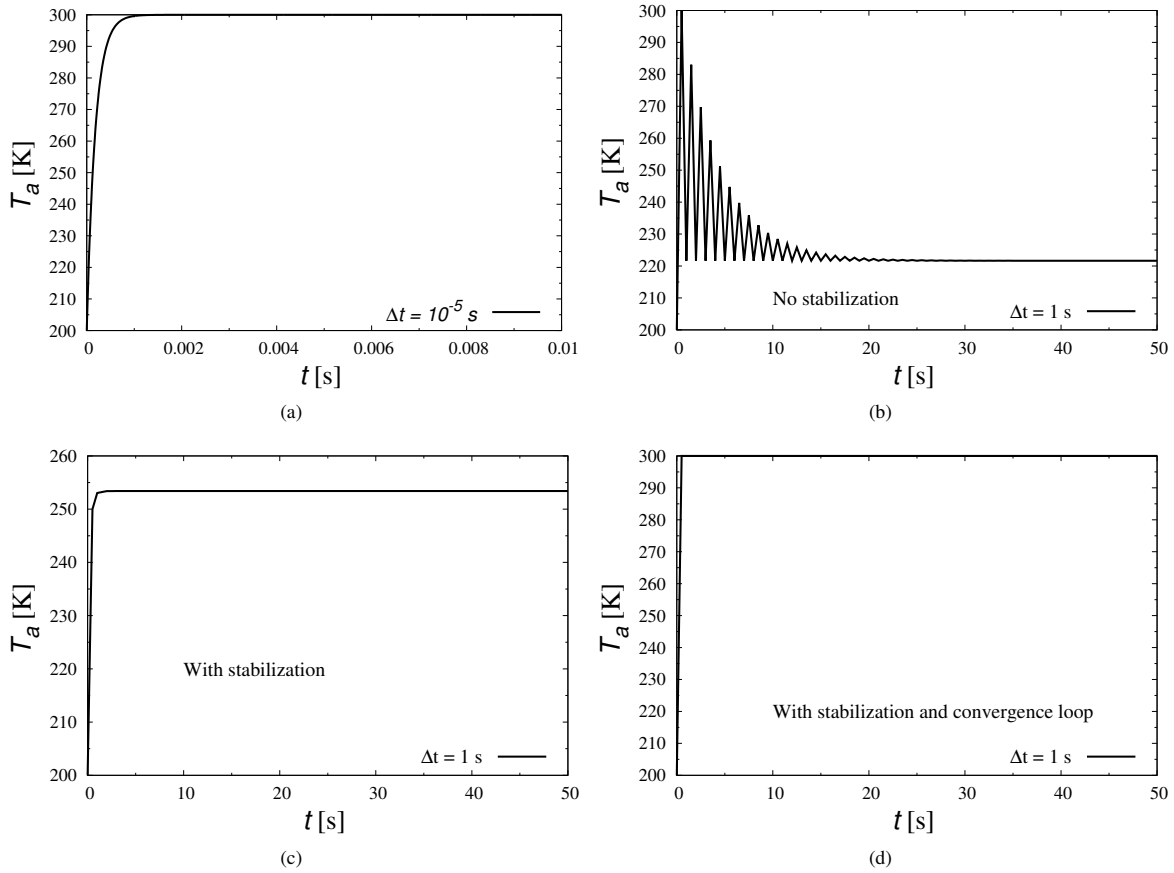

Figure 1: Time evolution of the air temperature: original scheme using (a) a time step of 10^{-5} s, (b) a time step of 1 s, proposed scheme with a time step of 1 s (c) without the inner convergence loop and (d) with the inner convergence loop.

The energy conservation problem was solved if we included an inner convergence loop for the energy equation at each time step, which iterates until the maximum variation of the volumetric average temperature in the whole domain is satisfied for all phases. The volumetric average temperature variation in a given iteration i was calculated by:

$$\overline{\Delta T_{\alpha(i)}}^{V} = \frac{\sum_{cell} V_{cell} \left| T^{(i)}_{\alpha,cell} - T^{(i-1)}_{\alpha,cell} \right|}{\sum_{cell} V_{cell}} \tag{5}$$

Figure 1(d) shows the time evolution of the air temperature. As noted, the temperature reached the equilibrium temperature in the first time step, showing that energy conservation is preserved and the scheme is quite robust.

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